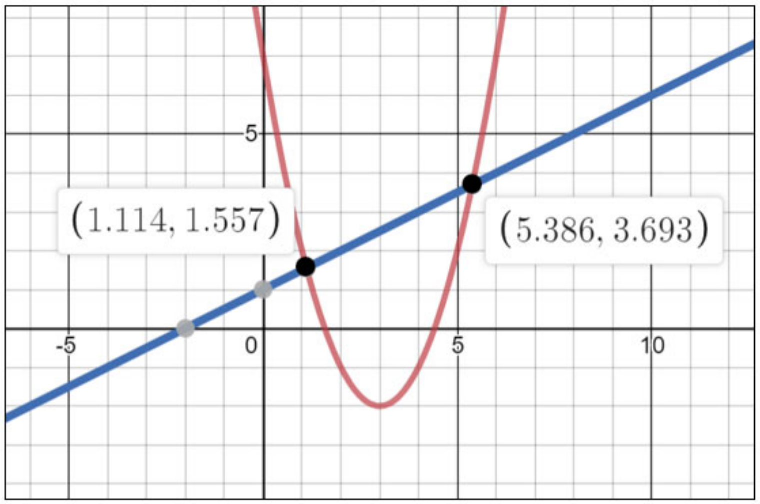
Objective: Solve Linear-Quadratic Systems Using Substitution

Concept

As you've learned, the graph of a linear-quadratic system cannot always be used to determine an exact solution. Using an algebraic method, such as substitution, to solve a linear-quadratic system you can always find exact solutions.



Note: Since the solution to a linear-quadratic system is any point of intersection of the graphs, it must be a real number. Therefore, imaginary solutions are not valid solutions.

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Concept

Steps to Solve a Linear-Quadratic System Using Substitution

1. **Solve the linear equation for x or y**. (It is often easier to solve for y in this step.)

2. **Substitute** the expression equal to x or y from step 1 **into the quadratic equation**.

3. **Solve the new equation** for the value of the variable.

4. If the value is valid (a real number), **find the value of the other variable**. If both values are valid, **write the solutions as ordered pairs**.

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Ex) Find the exact solutions using substitution

$$\begin{cases} 3x - y = -2 & \text{linear} \\ y = 5x^2 & \text{quad.} \end{cases}$$

① $3x - y = -2$
$+2 + y \quad +2 + y$
$y = 3x + 2$

② $y = 5x^2$

③ solve.
$3x + 2 = 5x^2$
$-3x - 2 \qquad -3x - 2$
$0 = 5x^2 - 3x - 2$
$0 = (5x + 2)(x - 1)$
$5x + 2 = 0$ or $x - 1 = 0$
$5x = -2 \qquad x = 1$
$x = \frac{-2}{5}$

④ find y's
*using $y = 3x + 2$

$x = -2/5 \qquad y = \frac{3}{1}\left(\frac{-2}{5}\right) + 2$
$= \frac{-6}{5} + \frac{2 \cdot 5}{1 \cdot 5}$
$= \frac{-6}{5} + \frac{10}{5} = \frac{4}{5}$

$x = 1 \qquad y = 3(1) + 2$
$3 + 2$
$= 5$

⑤ $\boxed{\text{solutions: } \left(\frac{-2}{5}, \frac{4}{5}\right) \text{ and } (1, 5)}$

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Ex) Find the exact solutions using substitution

$$\begin{cases} y+4=2(x+5)^2 & \text{quad.} \\ 3x-y=7 & \text{linear} \end{cases}$$

① $3x-y=7$

$-7+y \quad -7+y$

$y = 3x-7$

②

$y+4=2(x+5)^2$

$3x-7+4=2(x+5)^2$

$(x+5)(x+5)$

③ solve.

$3x-3=2(x^2+10x+25)$

$3x-3=2x^2+20x+50$

$-3x+3 \quad -3x \quad +3$

$0=2x^2+17x+53$

$a=2 \quad b=17 \quad c=53$

$x=\dfrac{-1(17)\pm\sqrt{(17)^2-[4(2)(53)]}}{2(2)}$

$53 \times 8 = 424$

$x=\dfrac{-17\pm\sqrt{289-424}}{4}$ imaginary

④ $\boxed{\text{no solution}}$

or $\boxed{\emptyset}$

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Ex) Find the exact solutions using substitution

$$\begin{cases} -x + y = 1 & \text{linear} \\ x^2 + y^2 = 25 & \text{quad.} \end{cases}$$

① $-x + y = 1$

$\quad +x \qquad\quad +x$

$y = x + 1$

② $x^2 + y^2 = 25$

$x^2 + (x+1)^2 = 25$

$(x+1)(x+1)$

③ solve.

$1x^2 + 1x^2 + 2x + 1 = 25$

$\qquad\qquad\quad -25 \quad -25$

$2x^2 + 2x - 24 = 0$

$2(x^2 + x - 12) = 0$

$2(x+4)(x-3) = 0$

$2 \neq 0 \qquad x + 4 = 0 \qquad x - 3 = 0$

$\qquad\qquad x = -4 \qquad x = 3$

④ find y's

☆ using $y = x + 1$

$x = -4, \quad y = -4 + 1 = -3$

$x = 3 \quad y = 3 + 1 = 4$

$\boxed{\text{⑤ solutions: } (-4, -3), (3, 4)}$

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Ex) Find the exact solutions using substitution

$$\begin{cases} x^2 + y^2 = 60 & \text{quad.} \\ y = -2x & \text{linear} \end{cases} \longrightarrow \textcircled{1}\ y = -2x$$

② $x^2 + y^2 = 60$

③ solve.

$x^2 + (-2x)^2 = 60$

$-2x \cdot -2x$

$1x^2 + 4x^2 = 60$

$\frac{5x^2}{5} = \frac{60}{5}$

$x^2 = 12$

$\sqrt{\cancel{x^2}} = \pm\sqrt{12}$

$\sqrt{4}\cdot\sqrt{3}$

$x = -2\sqrt{3},\ 2\sqrt{3}$

④ find y's

using $y = -2x$

$x = -2\sqrt{3}$

$y = -2 \cdot -2\sqrt{3} = 4\sqrt{3}$

$x = 2\sqrt{3}$

$y = -2 \cdot 2\sqrt{3} = -4\sqrt{3}$

⑤ solutions:

$(-2\sqrt{3}, 4\sqrt{3})$ and $(2\sqrt{3}, -4\sqrt{3})$

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Closure

Using your own words, explain why imaginary values are not valid solutions.

If the values are imaginary this means the graphs of the two equations do not intersect, so there is no solution to the problem.

Imaginary values are not valid solutions because they do not correspond to points on the x-y-coordinate plane.