Objective: Solve polynomial equations involving sine, cosine, and tangent.

## Concept

Recall: The three trigonometric functions of an angle $\theta$ (sine, cosine, and tangent) are defined as ratios of the sides of a right triangle.


SOHCAHTOA can be used as a device to help remember the definitions of sine, cosine, and tangent of an angle.

Objective: Solve polynomial equations involving sine, cosine, and tangent.
Ex) Find all real values of $x$ that satisfy the given situation. Round to two decimal places if necessary.


$$
\begin{aligned}
& \sin \theta=\frac{o p p}{h y p} \\
& \sin 35^{\circ}=\frac{5}{x^{2}-3}
\end{aligned}
$$

$$
\begin{aligned}
& \begin{array}{l}
\text { (2) solve } \\
\left(x^{2}-3\right) \cdot \sin 35^{\circ}=\frac{5}{x^{2}-3} \cdot\left(x^{2}-3\right)
\end{array} \\
& \frac{\left(x^{2}-3\right) \cdot \sin 35^{\circ}}{\sin 35^{\circ}}=\frac{5}{\sin 35^{\circ}}
\end{aligned}
$$

$$
\begin{aligned}
& x^{2}-3=\frac{5}{\sin 35^{\circ}}+3 \\
& x^{2}=\frac{5}{\sin 35^{\circ}}+3 \\
& \sqrt{x^{2}}= \pm \sqrt{\frac{5}{\sin 35^{\circ}}+3} \\
& x= \pm \sqrt{\left(\frac{5}{\sin \left(35^{\circ}\right)}+3\right)} \\
& x \approx-3.42,3.42
\end{aligned}
$$

Objective: Solve polynomial equations involving sine, cosine, and tangent.
Ex) Find all real values of $x$ that satisfy the given situation. Round to two decimal places if necessary.


$$
\star \cos \theta=\frac{a d j}{h y p}
$$

(1) $\cos 17^{\circ}=\frac{x^{2}}{9}$

$$
\begin{aligned}
& \text { (2) solve. } 17^{\circ}=\frac{x^{2}}{9} \cdot 9 \\
& 9 \cdot \cos 17^{\prime} \\
& 9 \cdot \cos 17^{\circ}=x^{2} \\
& \pm \sqrt{9 \cos 17^{\circ}}=\sqrt{x^{2}} \\
& x= \pm \sqrt{\left(9 \cdot \cos \left(17^{\circ}\right)\right)} \\
& x \approx-2.93,2.93
\end{aligned}
$$

Objective: Solve polynomial equations involving sine, cosine, and tangent.
Practice) Find all real values of $x$ that satisfy the given situation. Round to two decimal places if necessary.

side adjacent

$$
\begin{aligned}
& \cos 41^{\circ}=\frac{x^{2}+5}{19} \\
& (19) \cdot \cos 41^{\circ}=\frac{x^{2}+5}{19} \cdot(19) \\
& 19 \cdot \cos 41^{\circ}=x^{2}+5 \\
& \frac{-5-5}{-5} \\
& x^{2}=19 \cdot \cos \left(41^{\circ}\right)-5 \\
& x= \pm \sqrt{19 \cdot \cos \left(41^{\circ}\right)-5} \\
& x \approx-3.06,3.06
\end{aligned}
$$

Objective: Solve polynomial equations involving sine, cosine, and tangent.
Ex) Find all real values of $x$ that satisfy the given situation. Round to two decimal places if necessary.


$$
\# \tan \theta=\frac{o p p}{a d j}
$$

$$
\text { (1) } \tan 79^{\circ}=\frac{x^{2}+7}{8}
$$

(2) solve.

$$
\begin{aligned}
& \quad 8 \cdot \tan 79^{\circ}-7=x^{2} \\
& \pm \sqrt{8 \cdot \tan 79^{\circ}-7}=\sqrt{x^{k}} \\
& x=\quad \pm \sqrt{\left(8 \cdot \tan \left(79^{\circ}\right)-7\right)}
\end{aligned}
$$

$$
x \approx-5.84,5.84
$$

Objective: Solve polynomial equations involving sine, cosine, and tangent.
Practice) Find all real values of $x$ that satisfy the given situation. Round to two decimal places if necessary.


$$
\begin{aligned}
& \tan 50^{\circ}=\frac{7}{x^{2}} \\
& \left(x^{2}\right) \cdot \tan 50^{\circ}=\frac{7}{x^{2}} \cdot\left(x^{2}\right) \\
& x^{2} \cdot \tan 50^{\circ}=7 \\
& \frac{x^{2} \cdot \tan 50^{\circ}}{\tan 50^{\circ}}=\frac{7}{\tan 50^{\circ}} \\
& x^{2}=\frac{7}{\tan 50^{\circ}} \\
& x= \pm \sqrt{\frac{7}{\tan 50^{\circ}}} \\
& x \approx-2.42,2.42
\end{aligned}
$$

Objective: Solve polynomial equations involving sine, cosine, and tangent.

## Closure

Demetri solved the problem shown. He made two errors.
Explain his errors and then find the correct solution.


$$
\begin{aligned}
& \tan 16^{\circ}=\frac{12}{x^{2}-1} \\
& (12) \cdot \tan 16^{\circ}=\frac{12}{x^{2}-1} \cdot(1 \\
& 12 \cdot \tan 16^{\circ}=x^{2}-1 \\
& \quad \frac{+1}{+1} \\
& x^{2}=12 \cdot \tan \left(16^{\circ}\right)+1 \\
& x= \pm \sqrt{12 \cdot \tan \left(16^{\circ}\right)+1} \\
& x \approx-2.11,2.11
\end{aligned}
$$

Demetri's first error is he used tangent instead of sine. His second error is in the next step where he multiplied both sides by 12 instead of $x^{2}-1$. The correct solutions are about -6.67 and 6.67.
$\sin 16^{\circ}=\frac{12}{x^{2}-1}$
$\left(x^{2}-1\right) \cdot \sin 16^{\circ}=\frac{12}{x^{2}-1} \cdot\left(x^{2}-1\right)$
$\left(x^{2}-1\right) \cdot \sin 16^{\circ}=12$
$\frac{\left(x^{2}-1\right) \cdot \sin 16^{\circ}}{\sin 16^{\circ}}=\frac{12}{\sin 16^{\circ}}$
$x^{2}-1=\frac{12}{\sin 16^{\circ}}$

| +1 | +1 |
| :--- | :--- |

$x^{2}=\frac{12}{\sin 16^{\circ}}+1$
$x= \pm \sqrt{\frac{12}{\sin 16^{\circ}}+1}$
$x \approx-6.67,6.67$

