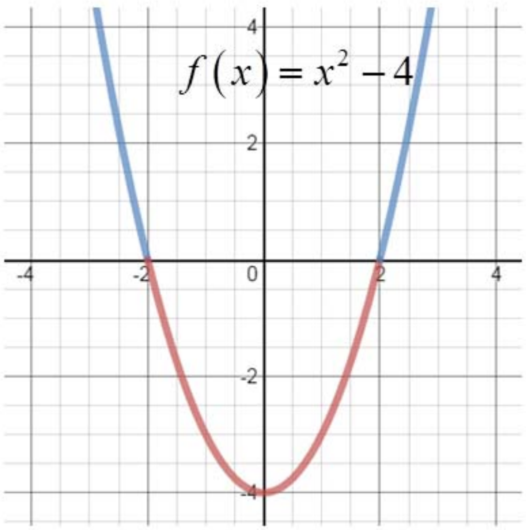
Objective: Solve a quadratic inequality without graphing

Concept

Solving a quadratic inequality algebraically uses a number line divided into intervals by the zeros of the related function. The intervals are tested to determine whether they would result in positive or negative values. The results correspond to where the related function would be positive or negative.



Find the solution to each quadratic inequality.

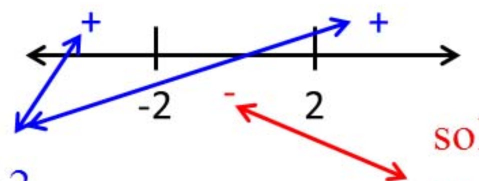
$Solve: x^2 - 4 > 0$

$(x+2)(x-2) > 0$

solution:

$x < -2$ or $x > 2$

$(-\infty, -2) \cup (2, \infty)$



$Solve: x^2 - 4 < 0$

$(x+2)(x-2) < 0$

solution:

$-2 < x < 2$

$(-2, 2)$

Objective: Solve a quadratic inequality algebraically.

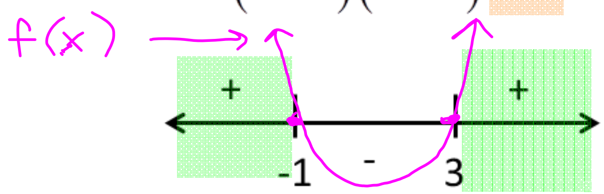
Concept

Find the solution to the quadratic inequality as an inequality and interval(s).

$$x^2 - 2x - 3 > 0$$

$$(x+1)(x-3) > 0$$

means positive function values



interval: $(-\infty, -1) \cup (3, \infty)$

inequality $x < -1$ or $x > 3$

Objective: Solve a quadratic inequality algebraically.

Concept

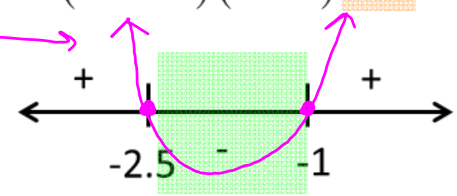
Find the solution to the quadratic inequality as an inequality and interval(s).

$$2x^2 + 7x + 5 < 0$$

$$(2x+5)(x+1) < 0$$

means negative function values

f(x)



interval: $(-2.5, -1)$

inequality: $-2.5 < x < -1$

Objective: Solve a quadratic inequality algebraically.

Concept

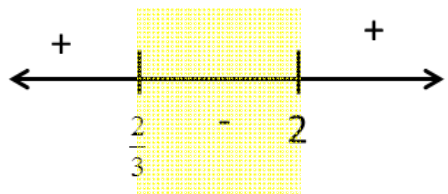
Find the solution to the quadratic inequality as an inequality and interval(s).

$$-3x^2+8x-4>0$$

$$3x^2-8x+4<0$$

$$(3x-2)(x-2)<0$$

means negative function values



$$\boxed{\begin{array}{l}\text{interval: } \left(\frac{2}{3}, 2\right) \\ \text{inequality: } \frac{2}{3} < x < 2\end{array}}$$

Objective: Solve a quadratic inequality algebraically.

Steps to Solve a Quadratic Inequality

1. Write the inequality in standard form, if necessary.
2. Set the inequality equal to zero and find the roots.
3. Create a number line that includes the roots.
4. Determine whether each interval on the number line will have positive or negative values. To do this, use a value from the interval for x and calculate the result.
5. Interpret the number line in terms of the inequality symbol. (< 0 *means the solution is the negative intervals*, > 0 *means the solution is the positive intervals*)
6. Write the solution using the specified notation.

Objective: Solve a quadratic inequality algebraically.

Ex) Solve the quadratic inequality algebraically. Write the solution as an inequality and an interval.

① $x^2 + 2x - 24 > 0$

② $x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x + 6 = 0 \qquad x - 4 = 0$

$\underline{-6 \quad -6} \qquad \underline{+4 \quad +4}$

roots/zeros $x = -6 \qquad x = 4$

③,④ use $(x+6)(x-4) > 0$ ⑤

solution

⑥ $\boxed{\begin{array}{l} \text{interval: } (-\infty, -6) \cup (4, \infty) \\ \text{inequality } \quad x < -6 \text{ or } x > 4 \end{array}}$

Objective: Solve a quadratic inequality algebraically.

Ex) Solve the quadratic inequality algebraically. Write the solution as an inequality and an interval.

1. $2x^2 + 11x + 12 < 0$

2. $2x^2 + 11x + 12 = 0$

$(2x+3)(x+4) = 0$

$2x + 3 = 0$ $\qquad$ $x + 4 = 0$

$-3 \quad -3$ $\qquad$ $-4 \quad -4$

$\frac{2x}{2} = \frac{-3}{2}$ $\qquad$ $x = -4$

$x = \frac{-3}{2}$ $\qquad$ roots/zeros

$x = -1\frac{1}{2}$

3,4 $\quad$ use $(2x+3)(x+4) < 0$ (5)

solution

5. $\boxed{\begin{array}{ll} \text{interval:} & (-4, -3/2) \\ \text{inequality} & -4 < x < \frac{-3}{2} \end{array}}$

Objective: Solve a quadratic inequality algebraically.

Ex) Solve the quadratic inequality algebraically. Write the solution as an inequality and an interval.

$$-3x^2+10x-8<0$$

① $-1\cdot[-3x^2+10x-8<0]$

$3x^2-10x+8>0$

② roots/zeros $3x^2-10x+8=0$

$(3x-4)(x-2)=0$

$3x-4=0$ $\quad$ $x-2=0$

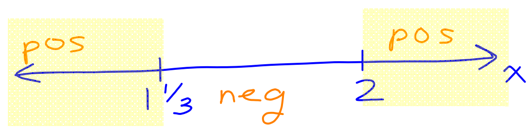
$+4 \quad +4$ $\quad$ $+2 \quad +2$

$3x=4$ $\quad$ $x=2$

$\frac{3x}{3}=\frac{4}{3}$

$x=\frac{4}{3}=1\frac{1}{3}$

(3,4) use $(3x-4)(x-2)>0$ ⑤



solution

⑥ interval $(-\infty,1\frac{1}{3})\cup(2,\infty)$

inequality $x<1\frac{1}{3}$ or $x>2$

Objective: Solve a quadratic inequality algebraically.

Closure

When solving a quadratic inequality algebraically, when would the solution use the positive intervals?

The solution would use the positive intervals when the inequality symbol is greater than zero in the factored form.