Objective: Use Exponential Models to Solve Context Problems

Concept

base 1·t

An **exponential growth function** has the form $f(t) = a(1 + r)^t$, where ***a* is the initial value** $(a > 0)$ and ***r* is the constant percent increase** (expressed as a decimal) for each unit increase in time t.

The **base, $1 + r$**, of an exponential growth function is called the growth factor.

> 1

The constant percent increase ***r* is called the growth rate.**

base < 1

1·t

An **exponential decay function** has the form $f(t) = a(1 - r)^t$, where ***a* is the initial value** $(a > 0)$ and ***r* is the constant percent decrease** (expressed as a decimal) for each unit increase in time t.

The **base $1 - r$** of an exponential decay function is called the decay factor.

The constant percent decrease ***r* is called the decay rate.**

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Concept

$r = ?$

Finding the Growth Rate or Decay Rate of an Exponential Function

base > 1

For a Growth Model: $\boldsymbol{f(x) = a(1+r)^t}$ where r is the growth rate.

Set $1 + r$ equal to the base value of the function model and solve for r. Rewrite as a percentage.

base < 1

For a Decay Model: $\boldsymbol{f(x) = a(1-r)^t}$ where r is the decay rate.

Set $1 - r$ equal to the base value of the function model and solve for r. Rewrite as a percentage.

In an Exponential Growth Model, $\underbrace{1 + r}_{\text{base}}$ is greater than/less than 1.

In an Exponential Decay Model, $\underbrace{1 - r}_{\text{base}}$ is greater than/less than 1.

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Ex) The population of an island over a period of time is modeled by the function $P(t) = 20{,}000(1.105)^t$, where t is in decades. At what rate is the island population changing? Is the population increasing or decreasing?

$r = ?$

$P(t) = 20{,}000(\underbrace{1.105}_{\text{base} > 1})^t$

base > 1 → growth

$1 + r = \text{base}$

$1 + r = 1.105$

$-1 \quad -1$

$r = 0.105$

$\times 100$

$= 10.5\%$

convert to %

- The population is changing at a rate of 10.5% per decade.
- The population is increasing.

Ex) Upon reaching a certain age, a person's hearing can be modeled by the function $H(t) = 5000(0.991)^t$, where t is years after a certain age. Determine the rate at which a person's hearing is increasing or decreasing after a certain age.

$r = ?$

$H(t) = 5000(0.991)^t$

base < 1 → decay

$1 - r = \text{base}$

$1 - r = 0.991$

$-.991 + r \quad -.991 + r$

$0.009 = r$

$\times 100$

0.9%

convert to %

A person's hearing is decreasing at a rate of 0.9% per year after a certain age.

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Ex) A culture of bacteria begins with 350 bacteria and increases by 1.4% per day.

a
a) Write a function $B(d)$ to model the bacteria population after d days.

exponent

growth ⇒ base = 1+r

① rate = 1.4% convert to decimal

÷100

r = 0.014 →

② base = 1 + 0.014
= 1.014

③ model

$B(d) = a \cdot (\text{base})^d \rightarrow \boxed{B(d) = 350(1.014)^d}$

starting amount when d = 0

b) Determine the expected population size of the culture after 42 hours.

Model: $B(d) = 350(1.014)^d$

* convert 42 hr to days

$B(d) = ?$ when $d = \frac{42}{24}$ days

$\frac{42\cancel{hr}}{1} \cdot \frac{1\,day}{24\cancel{hr}}$

$B\left(\frac{42}{24}\right) = 350(1.014)^{42/24} \approx 359$ bacteria

calc. 350 × 1.014 ^ (42 ÷ 24) =

After 42 hours it's expected there will be about 359 bacteria in the culture.

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Objective: Use Exponential Models to Solve Context Problems

Ex) The value of a truck purchased new for $28,000 in 2015 depreciates by 9.5% each year.

a

"decreases"

a) Write a function $V(t)$ to model the value of the truck where t is in years since 2015.

exponent

decay → base = 1 − r

① rate 9.5% ÷ 100 ← convert to decimal

$r = 0.095$

② base = 1 − r
= 1 − 0.095
= 0.905

③ model

$V(t) = a \cdot (\text{base})^t$

↑ beginning amount when $t = 0$

→ $\boxed{V(t) = 28{,}000(0.905)^t}$

b) Estimate how much the truck will be worth in 2027.

Use $V(t) = 28{,}000(0.905)^t$

$V(t) = ?$ when $t = 2027 - 2015$
$t = 12$ yrs

$V(12) = 28000(0.905)^{12}$
$= \$8451.64$

In 2027 the truck will be worth an estimated $8451.64.

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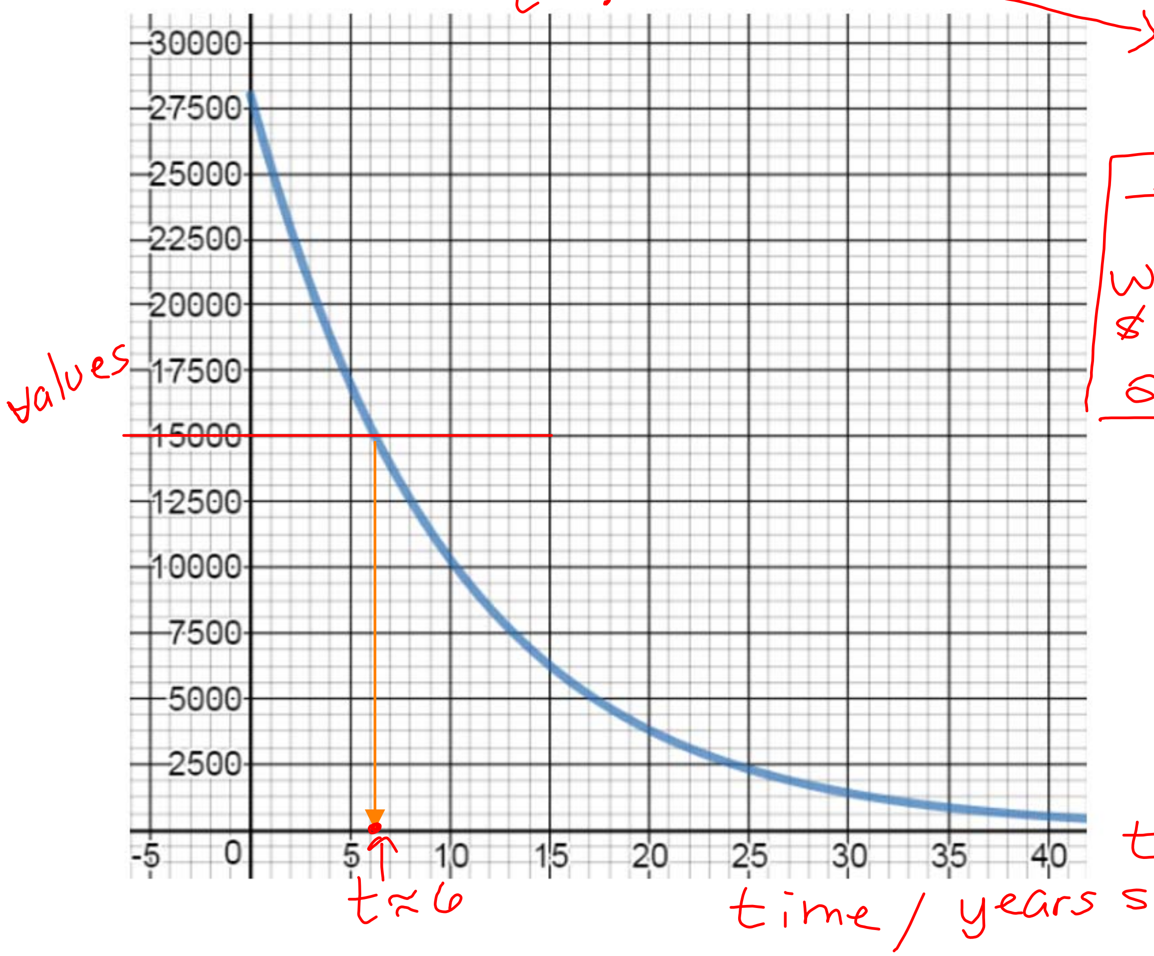
calendar year

c) Use the graph to estimate when the truck will be worth $15,000.

$t = ?$ → $2015 + 6$
$= 2021$



The truck will be worth $15,000 around 2021.