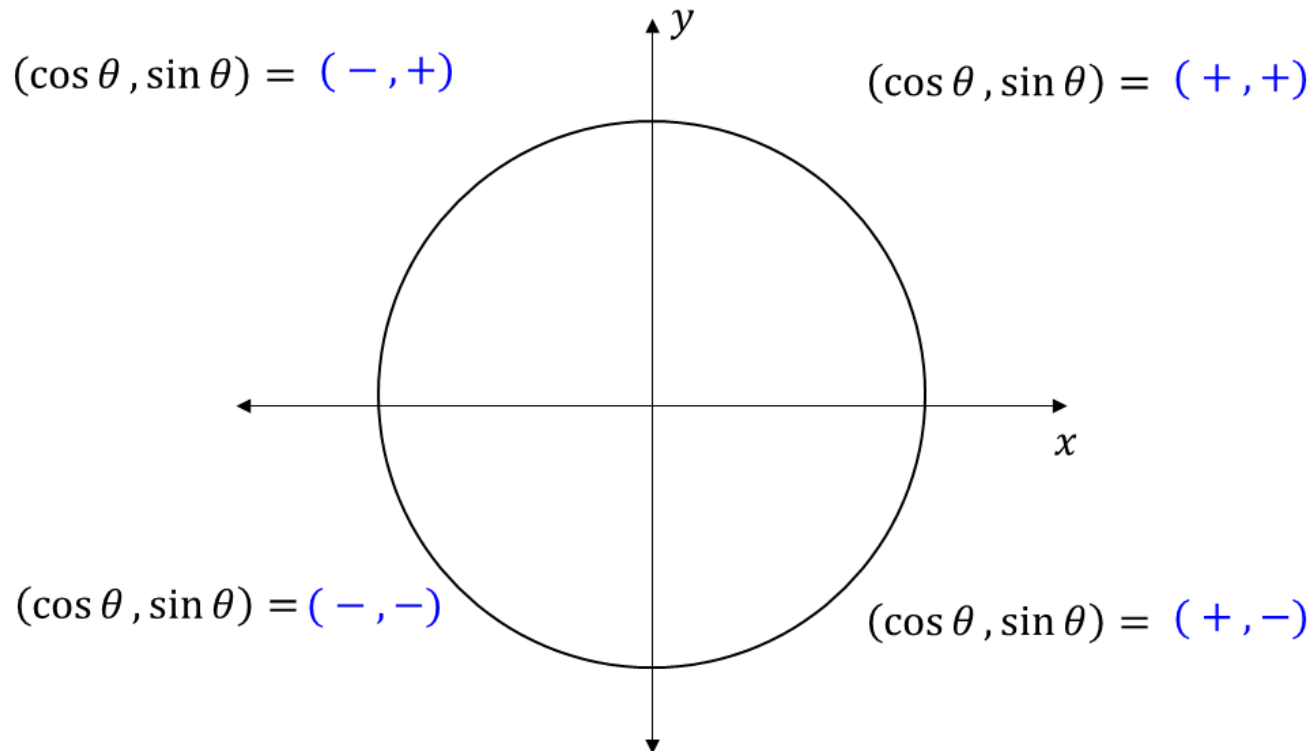


Objective: Find Trigonometric Values of Unit Circle Angles

Prior Knowledge

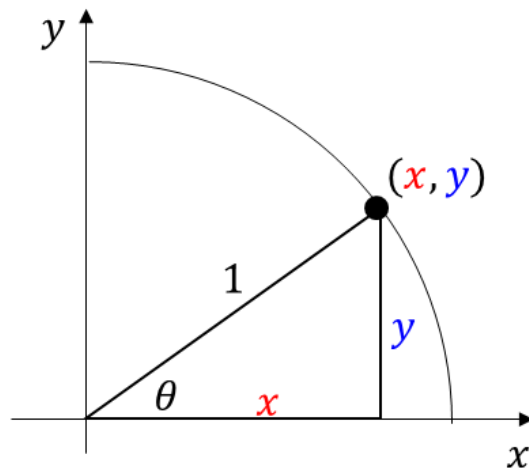
Identify whether sine and cosine are positive or negative in each quadrant.



## Objective: Find Trigonometric Values of Unit Circle Angles

Concept**Quotient Identities**

You have learned that in the Unit Circle, the cosine and sine of the angle correspond to the  $x$ - coordinate and  $y$ - coordinate, respectively, where the terminal side of the angle intersects the Unit Circle. Using this and your knowledge of Right Triangle Trigonometry, we can derive what are known as the Quotient Identities.



$$1. \sin \theta = \frac{y}{1} = y \text{ and } \cos \theta = \frac{x}{1} = x$$

$$2. \tan \theta = \frac{y}{x}$$

3. Substitution for  $x$  and  $y$  in the tangent statement results in  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ .

4.  $\tan \theta = \frac{\sin \theta}{\cos \theta}$  is what is known as a **Quotient Identity**. Since tangent and cotangent are reciprocals, **another Quotient Identity** is  $\cot \theta = \frac{\cos \theta}{\sin \theta}$ .

Objective: Find Trigonometric Values of Unit Circle Angles

Concept

**Quotient Identities**

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

**Reciprocal Identities**

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

Objective: Find Trigonometric Values of Unit Circle Angles

Ex) Use the Unit Circle and Trigonometric Identities to find each exact value.

<div style="border: 1px solid blue; padding: 5px; display: inline-block; margin-bottom: 10px;"> <math>\sec \frac{\pi}{6} = \frac{2\sqrt{3}}{3}</math> </div> $= \frac{1}{\cos \frac{\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}}$ $= 1 \cdot \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{2\sqrt{3}}{3}$	<div style="border: 1px solid blue; padding: 5px; display: inline-block; margin-bottom: 10px;"> <math>\sec \frac{3\pi}{4} = -\sqrt{2}</math> </div> $= \frac{1}{\cos \frac{3\pi}{4}} = \frac{1}{-\frac{\sqrt{2}}{2}}$ $= 1 \cdot \frac{-2}{\sqrt{2}} = \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ $= \frac{-2\sqrt{2}}{2} = -\sqrt{2}$	<div style="border: 1px solid blue; padding: 5px; display: inline-block; margin-bottom: 10px;"> <math>\sec \frac{\pi}{2} = \text{undefined}</math> </div> $= \frac{1}{\cos \frac{\pi}{2}} = \frac{1}{0}$ $= \text{undefined}$
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Objective: Find Trigonometric Values of Unit Circle Angles

Ex) Use the Unit Circle and Trigonometric Identities to find each exact value.

$$\boxed{\csc \frac{\pi}{3} = \frac{2\sqrt{3}}{3}}$$

$$= \frac{1}{\sin \frac{\pi}{3}} = \frac{1}{\frac{\sqrt{3}}{2}}$$

$$= 1 \cdot \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{3}$$

$$\boxed{\csc \frac{5\pi}{4} = -\sqrt{2}}$$

$$= \frac{1}{\sin \frac{5\pi}{4}} = \frac{1}{-\frac{\sqrt{2}}{2}}$$

$$= 1 \cdot \frac{-2}{\sqrt{2}} = \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-2\sqrt{2}}{2} = -\sqrt{2}$$

$$\boxed{\csc \frac{\pi}{2} = 1}$$

$$= \frac{1}{\sin \frac{\pi}{2}} = \frac{1}{1}$$

$$= 1$$

Objective: Find Trigonometric Values of Unit Circle Angles

Ex) Use the Unit Circle and Trigonometric Identities to find each exact value.

\*  $\tan \theta = \frac{\sin \theta}{\cos \theta}$

$$\tan \frac{3\pi}{4} = -1$$

$$= \frac{\sin \frac{3\pi}{4}}{\cos \frac{3\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{-2}{\sqrt{2}} = \frac{-2\sqrt{2}}{2\sqrt{2}}$$

$$= -1$$

$$\tan \frac{7\pi}{6} = \frac{\sqrt{3}}{3}$$

$$= \frac{\sin \frac{7\pi}{6}}{\cos \frac{7\pi}{6}} = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$$

$$= \frac{-1}{2} \cdot \frac{-2}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\tan \frac{\pi}{2} = \text{undefined}$$

$$= \frac{\sin \frac{\pi}{2}}{\cos \frac{\pi}{2}} = \frac{1}{0}$$

$$= \text{undefined}$$

Objective: Find Trigonometric Values of Unit Circle Angles

Ex) Use the Unit Circle and Trigonometric Identities to find each exact value.

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\cot \frac{\pi}{6} = \sqrt{3}$$

$$= \frac{\cos \frac{\pi}{6}}{\sin \frac{\pi}{6}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$$

$$\cot \frac{5\pi}{3} = -\frac{\sqrt{3}}{3}$$

$$= \frac{\cos \frac{5\pi}{3}}{\sin \frac{5\pi}{3}} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{2} \cdot \frac{-2}{\sqrt{3}} = \frac{-1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\cot \frac{\pi}{2} = 0$$

$$= \frac{\cos \frac{\pi}{2}}{\sin \frac{\pi}{2}} = \frac{0}{1} = 0$$

## Objective: Find Trigonometric Values of Unit Circle Angles

### Closure

Explain when each of the trigonometric values will be undefined:

cosecant: Cosecant will be undefined when sine equals 0, at  $0$ ,  $\pi$  and  $2\pi$ .

secant: Secant will be undefined when cosine equals 0, at  $\frac{\pi}{2}$  and  $\frac{3\pi}{2}$ .

tangent: Tangent will be undefined when cosine equals 0, at  $\frac{\pi}{2}$  and  $\frac{3\pi}{2}$ .

cotangent: Cotangent will be undefined when sine equals 0, at  $0$ ,  $\pi$  and  $2\pi$ .

