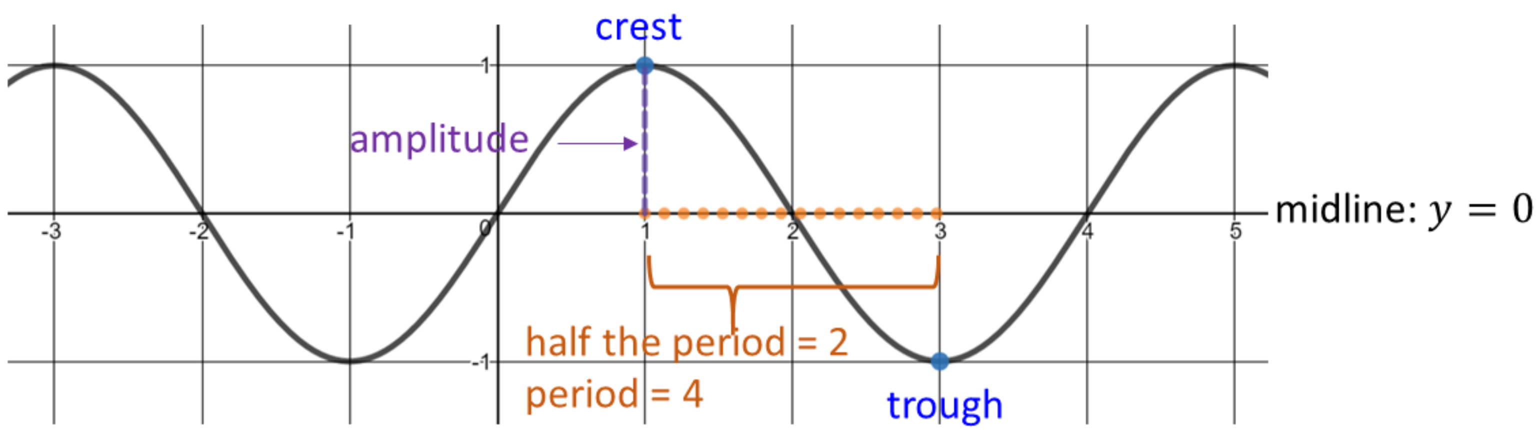
Objective: Model periodic phenomena with trigonometric functions

Concept

Things that move in a circular or wavelike motion (periodic phenomena) can be modeled using a sine or cosine function. The wave-like shape of the sine and cosine functions have a "crest" (where the maximum occurs) and a "trough" (where the minimum occurs).

- Halfway between the "crest" and "trough" is the graph's **midline**.
- The distance that the "crest" rises above the midline or the distance the "trough" falls below the midline is the **amplitude**.
- The horizontal distance between the "crest" and "trough" is **half the period**.



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Concept

When modeling periodic phenomena either a sine or cosine function can be used since performing a horizontal translation of $\frac{\pi}{2}$ units on sine produces cosine. If the type of function to be used isn't specified, you can use these general rules to decide which type of function (sine or cosine) would be easier to create.

- The y-intercept is on the midline: use $f(x) = a\sin\left(\frac{1}{b}x\right) + k$
- The y-intercept is a maximum (crest) or minimum (trough): use $f(x) = a\cos\left(\frac{1}{b}x\right) + k$
- The y-intercept is none of the above: use $f(x) = a\sin\left(\frac{1}{b}x + c\right) + k$ or $f(x) = a\cos\left(\frac{1}{b}x + c\right) + k$

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Steps to Write a Sine or Cosine Model

of the Form $f(x) = a sin\frac{1}{b}x + k$ or $f(x) = a\ cos\frac{1}{b}x + k$

1. Determine the function to be used. Sketch a graph if necessary.
2. Find the midline, value of k, which is halfway between the maximum and minimum values.
3. Find the amplitude, $|a|$. Determine whether a is positive or negative.
4. Find the period, P.
5. Find the value of $\frac{1}{b}$ using $P = \frac{2\pi}{\frac{1}{b}}$
6. Write the function.

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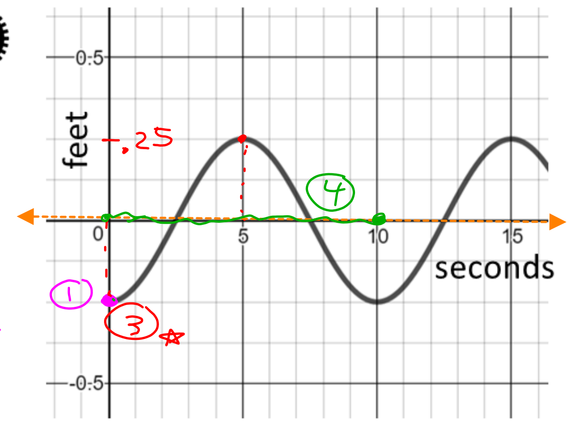
Ex) As a gear spins, the height of a point on its edge, above or below the horizontal diameter of the gear, can be modeled by the graph shown. Write a function that models this situation.



① use $f(x) = a\cos\left(\frac{1}{b}x\right) + k$

② midline $y = 0 \rightarrow k = 0$

③ amplitude $= 0.25 \rightarrow a = -0.25$ ☆ x-axis reflection

④ $P = 10$

⑤ $P = \dfrac{2\pi}{\frac{1}{b}}$ $\quad 10 = \dfrac{2\pi}{\frac{1}{b}} \rightarrow \dfrac{10 \cdot \frac{1}{b}}{10} = \dfrac{2\pi}{10}$

☆ $\dfrac{1}{b} = \dfrac{\pi}{5}$

⑥ $\boxed{f(x) = -0.25\cos\left(\frac{\pi}{5}x\right)}$

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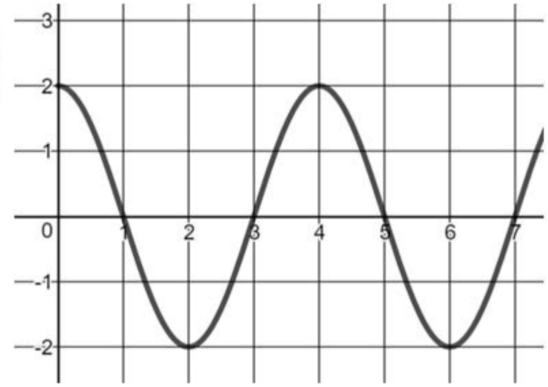
Practice) The height of a point on the edge of a spinning prize wheel relative to the horizontal diameter of the wheel can be modeled by the graph shown. Write a function for the graph.



1. the y-intercept is a maximum: use $f(x) = a\cos\frac{1}{b}x + k$

2. midline: $y = k \rightarrow y = 0 \rightarrow k = 0$

3. amplitude $= 2 \rightarrow$ since the y-intercept is a maximum, a is positive $\rightarrow a = 2$

4. half the period $= 2$
   period $= 4$

5. $P = \dfrac{2\pi}{\frac{1}{b}} \rightarrow 4 = \dfrac{2\pi}{\frac{1}{b}} \rightarrow \frac{1}{b}(4) = 2\pi \rightarrow \frac{1}{b} = \frac{2\pi}{4} \rightarrow \frac{1}{b} = \frac{\pi}{2}$

6. $\boxed{f(x) = 2\cos\frac{\pi}{2}x}$

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How can you determine the amplitude of a function by looking at its graph?

Find half the vertical distance between its maximum point (crest) and its minimum point (trough), or find the distance from its midline to either its maximum or minimum point.

How can you determine the period of a function by looking at its graph?

Determine the horizontal length of one cycle, or find the horizontal distance between the maximum (crest) and minimum (trough) and multiply by 2.

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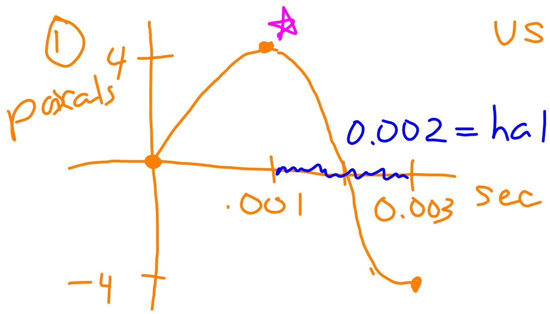
Ex) As a tuning fork vibrates it creates fluctuations in air pressure. The maximum change in air pressure, typically measured in pascals, is the sound wave's amplitude. At 0.001 seconds a soundwave crests at 4 pascals and at 0.003 seconds the soundwave troughs at −4 pascals. Write a function that models this situation if the sound was measured beginning at 0 pascals.



0.002 = half the period

use $f(x) = a\sin\left(\frac{1}{b}x\right) + k$

② midline $y = 0 \rightarrow k = 0$

③ amplitude $= 4 \rightarrow a = 4$

no reflection

④ $P = 2 \cdot 0.002$

$P = 0.004$

⑤ $P = \dfrac{2\pi}{\frac{1}{b}}$

$0.004 = \dfrac{2\pi}{\frac{1}{b}}$

$\dfrac{0.004 \cdot \frac{1}{b}}{0.004} = \dfrac{2\pi}{0.004}$

$\dfrac{1}{b} = \dfrac{\pi}{0.002}$

⑥ $\boxed{f(x) = 4\sin\left(\frac{\pi}{0.002}x\right)}$

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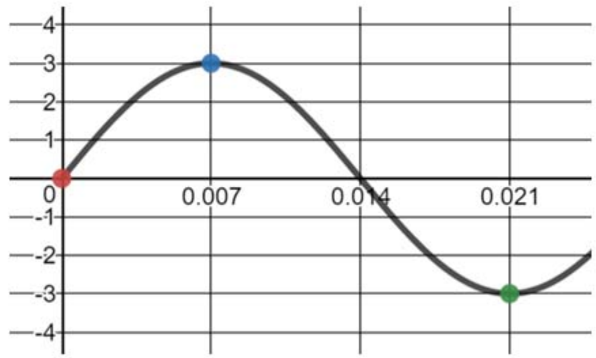
Practice) As a sound wave fluctuates, the maximum change in air pressure, typically measured in pascals, is the sound wave's amplitude. At 0.007 seconds a soundwave crests at 3 pascals. At 0.021 seconds the soundwave troughs at -3 pascals. Write a function that models this situation if the sound was measured beginning at 0 pascals.



1. the y-intercept is on the midline: use $f(x) = a\sin\frac{1}{b}x + k$
2. midline: $y = k \to y = 0 \to k = 0$
3. amplitude $= 3 \to$ since the maximum is reached first a is positive $\to a = 3$
4. half the period $= 0.0021 - 0.007 = 0.014$
   period $= 0.028$
5. $P = \dfrac{2\pi}{\frac{1}{b}} \to 0.028 = \dfrac{2\pi}{\frac{1}{b}} \to \dfrac{1}{b}(0.028) = 2\pi \to \dfrac{1}{b} = \dfrac{2\pi}{0.028} \to \dfrac{1}{b} = \dfrac{\pi}{0.014}$
6. $\boxed{f(x) = 3\sin\dfrac{\pi}{0.014}x}$

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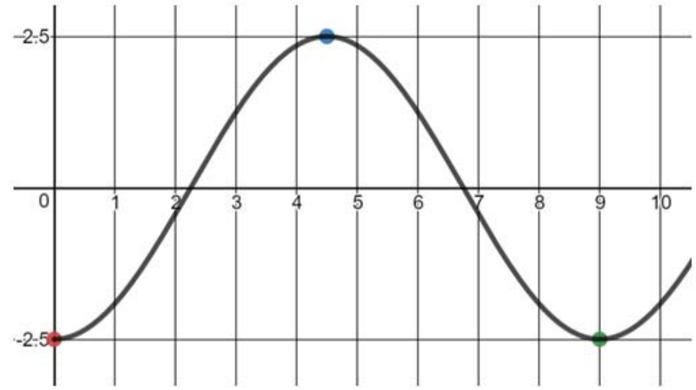
Practice) An object oscillates 5 feet from its minimum height to its maximum height. The object is back at the maximum height every 9 seconds. Write a function that can be used to model the height of the object if the object starts at its minimum height.



1. the y-intercept is a minimum: use $f(x) = a\cos\frac{1}{b}x + k$

2. $midline: y = k \rightarrow y = 0 \rightarrow k = 0$

3. amplitude $= \frac{5}{2} = 2.5 \rightarrow$ since the y-intercept is a minimum,

a is negative $\rightarrow a = -2.5$

4. period $= 9$ seconds

5. $P = \frac{2\pi}{\frac{1}{b}} \rightarrow 9 = \frac{2\pi}{\frac{1}{b}} \rightarrow \frac{1}{b}(9) = 2\pi \rightarrow \frac{1}{b} = \frac{2\pi}{9} \rightarrow \frac{1}{b} = \frac{2\pi}{9}$

6. $\boxed{f(x) = -2.5\cos\frac{2\pi}{9}x}$

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How can being given the crest and trough of a function help you find its period?

The horizontal distance between the crest and trough of a function is half a cycle (half the period), so twice this distance is the period.

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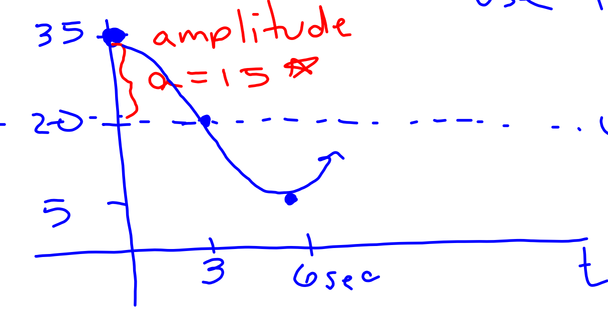
Ex) Water turns a water wheel at an old mill. At a time t, in seconds, a point on the wheel has a height h relative to the stream into which the water falls. In 6 seconds a point on the wheel travels from a maximum height of 35 feet to a minimum of 5 feet. Write a function that models this situation if the point starts at its maximum height.

use $f(x) = a\cos\left(\frac{1}{b}x\right) + k$

(1)



$y = 20 \rightarrow k = 20$

(4) $\boxed{f(x) = 15\cos\left(\frac{\pi}{6}x\right) + 20}$

(2) period $= 2 \cdot 6\text{sec}$

$= 12\text{sec}$

(3) $P = \dfrac{2\pi}{\frac{1}{b}}$ $\quad 12 = \dfrac{2\pi}{\frac{1}{b}} \rightarrow \dfrac{12 \cdot \frac{1}{b}}{12} = \dfrac{2\pi}{12} \rightarrow \dfrac{1}{b} = \dfrac{\pi}{6}$

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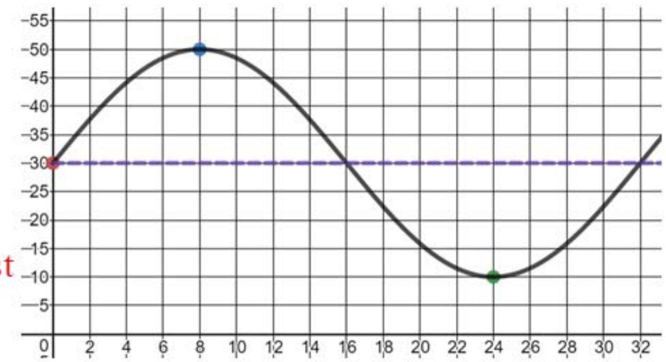
Practice) The motion of a gondola car on a Ferris wheel is periodic. On a particular Ferris wheel a gondola car starts at a height of 30 feet, reaches a maximum height above the ground of 50 feet and then a minimum height of 10 feet. It takes 32 seconds for the gondola car to return to its starting position. Write a function that can model this situation.



1. the y-intercept is on the midline: use $h(t) = a\sin\frac{1}{b}t + k$

2. midline: $y = k \rightarrow k = \frac{50-10}{2} + 10 \rightarrow y = 20 + 10 \rightarrow k = 30$

3. amplitude $= \frac{50-10}{2} = 20 \rightarrow$ since the maximum is reached first

a is positive $\rightarrow a = 20$

4. period = 32 seconds

5. $P = \frac{2\pi}{\frac{1}{b}} \rightarrow 32 = \frac{2\pi}{\frac{1}{b}} \rightarrow \frac{1}{b}(32) = 2\pi \rightarrow \frac{1}{b} = \frac{2\pi}{32} \rightarrow \frac{1}{b} = \frac{\pi}{16}$

6. $\boxed{h(t) = 20\sin\frac{\pi}{16}t + 30}$

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Closure

Explain how you know when to use a sine model and when to use a cosine model for periodic phenomena.

Use a sine model when the initial point is on the midline and use a cosine model when the initial point is a maximum or minimum.