Objective: Solve Base e Exponential Equations.

Concept

Steps to Solve a Base e Exponential Equation of the Form $e^{ax+b} = c$

1. Take the **natural logarithm** of both sides.
2. Use the **Power Property of Logarithms**: $\log_b a^x = x log_b a$
3. Solve this equation using algebra.
4. State the solution to the exponential equation. Approximate if necessary.

Property of Logarithms

Given $\log_b b = 1$

Then $\mathbf{\ln e = 1}$

Objective: Solve Base e Exponential Equations.

Ex) Solve the equation. Give the exact solution and the approximate solution to three decimal places.

$$\frac{10}{5} = \frac{5e^{4x}}{5}$$

(1) $2 = e^{4x}$

(2) $\ln 2 = \ln e^{4x}$

(3) $\ln 2 = 4x \cdot \ln e \rightarrow \log_e e = 1$

(4) $\ln 2 = 4x \cdot 1$

$$\frac{\ln 2}{4} = \frac{4x}{4}$$

(5) $x = \frac{\ln 2}{4}$

$x \approx 0.173$

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Ex) Solve the equation. Give the exact solution and the approximate solution to three decimal places.

$$-3e^{x+2} - 8 = -17$$

① $+8 \quad +8$

$$\frac{-3e^{x+2}}{-3} = \frac{-9}{-3}$$

$$e^{x+2} = 3$$

② $\ln e^{x+2} = \ln 3$

③ $(x+2)\cdot \ln e = \ln 3$

④ $(x+2)\cdot 1 = \ln 3$

$$x + 2 = \ln 3$$

$-2 \quad -2$

⑤ $x = \ln(3) - 2$

$x \approx -0.901$

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Concept

When you **have** an equation with **a trinomial structure, first consider how the trinomial might be factored into two binomials**. If this is possible, you can then use the Zero Product Property and solve for the variable from there.

Ex) Solve the equation. Give the exact solution and the approximate solution to three decimal places.

$$e^{2x} - 5e^x + 6 = 0$$

① $(e^x - 3)(e^x - 2) = 0$

② $e^x - 3 = 0$ or $e^x - 2 = 0$

③ $+3 \quad +3$; $+2 \quad +2$

$e^x = 3$; $e^x = 2$

④ $\ln e^x = \ln 3$; $\ln e^x = \ln 2$

$x \cdot \ln e = \ln 3$; $x \cdot \ln e = \ln 2$

$x \cdot 1 = \ln 3$; $x \cdot 1 = \ln 2$

⑤ $x = \ln 3$; $x = \ln 2$

$x \approx 1.099$; $x \approx 0.693$

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Ex) Solve the equation. Give the exact solution and the approximate solution to three decimal places.

$$2e^{2x} + 3e^{x} - 5 = 0$$

① $(2e^x + 5)(e^x - 1) = 0$

② $2e^x + 5 = 0$ or $e^x - 1 = 0$

③ $-5 \quad -5$ $\qquad +1 \quad +1$

$\frac{2e^x}{2} = \frac{-5}{2}$ $\qquad e^x = 1$

$e^x = -\frac{5}{2}$

④ $\ln e^x = \ln 1$

$x \cdot \ln e = \ln 1$

$x \cdot 1$

$x = \ln 1$

$x = 0$

④ $\ln e^x = \ln\left(\frac{-5}{2}\right)$

$x \cdot \ln e = \ln\left(\frac{-5}{2}\right)$

$x \cdot 1$

$x = \ln\left(\frac{-5}{2}\right)$

not valid

Objective: Solve Base e Exponential Equations.

Ex) Solve the equation. Give the exact solution and the approximate solution to three decimal places.

$$e^{2x} - 5e^x = 0$$

$e^x \cdot e^x$

① gcf $e^x(e^x - 5) = 0$

② $e^x = 0$ or $e^x - 5 = 0$

$+5 \quad +5$

$e^x = 5$

③ $\ln e^x = \ln 0$

$\ln e^x = \ln 5$

④ $x \cdot \ln e = \ln 0$

$x \cdot 1$

$x = \ln 0$

not valid

④ $x \cdot \ln e = \ln 5$

$x \cdot 1$

⑤ $x = \ln 5$

$x \approx 1.609$

Objective: Solve Base e Exponential Equations.

Practice) Solve the equation. Give the exact solution and the approximate solution to three decimal places.

$$2e^{2x} - e^{x} = 0$$

$$e^{x}\left(2e^{x} - 1\right) = 0$$

$$e^{x} = 0 \quad or \quad 2e^{x} - 1 = 0$$

$$e^{x} = \frac{1}{2}$$

$$\ln e^{x} = \ln 0 \qquad \ln e^{x} = \ln \frac{1}{2}$$

$$x \ln e = \ln 0 \qquad x \ln e = \ln \frac{1}{2}$$

$$\cancel{x = \ln 0} \qquad x = \ln \frac{1}{2}$$

$$\boxed{\begin{array}{l} x = \ln \frac{1}{2} \\ x \approx -0.693 \end{array}}$$

Objective: Solve Base e Exponential Equations.

Practice) Solve the equation. Give the exact solution and the approximate solution to three decimal places.

$$e^{4x} - 8e^{2x} + 15 = 0$$

$$\left(e^{2x} - 5\right)\left(e^{2x} - 3\right) = 0$$

$$e^{2x} - 5 = 0 \quad or \quad e^{2x} - 3 = 0$$

$$e^{2x} = 5 \qquad e^{2x} = 3$$

$$\ln e^{2x} = \ln 5 \qquad \ln e^{2x} = \ln 3$$

$$2x \ln e = \ln 5 \qquad 2x \ln e = \ln 3$$

$$x = \frac{\ln 5}{2} \qquad x = \frac{\ln 3}{2}$$

$$x = \frac{\ln 5}{2} \qquad x = \frac{\ln 3}{2}$$

$$x \approx 0.805 \qquad x \approx 0.549$$

Objective: Solve Base e Exponential Equations.

Closure

A student solved the equation $e^{4x} - 6 = 10$ as shown. Explain the student's mistake and find the correct solution.

$e^{4x} = 16$

$\ln e^{4x} = \ln 16$

$4x \cdot \ln e = \ln 16$

$4x \cdot 1 = \ln 16$

$\frac{4x}{4} = \frac{\ln 16}{4}$

$x = \ln 4$

$x = \ln 4$

$x \approx 1.386$

$e^{4x} = 16$

$\ln e^{4x} = \ln 16$

$4x \cdot \ln e = \ln 16$

$4x \cdot 1 = \ln 16$

$\frac{4x}{4} = \frac{\ln 16}{4}$

$x = \frac{\ln 16}{4}$

$x \approx 0.693$

The student divided the argument of 16 by 4, which is incorrect. The correct solution is $x = \frac{\ln 16}{4}$ or $x \approx 0.693$.