# Objective: Solving Context Problems With Logarithmic Models

## Concept

Logarithmic models can be used to represent many real-world situations. These models often use a base 10 or base $e$ logarithm. A calculator is often needed to evaluate the function model for the conditions of the problem.

## Approximating the Value of Logarithms Using a Scientific Calculator

The **base 10 logarithm** is called the **Common Logarithm**: $log_{10}x = \mathbf{log}\, \boldsymbol{x}$

The **base $e$ logarithm** is called the **Natural Logarithm**: $log_e x = \mathbf{ln}\, \boldsymbol{x}$

For example: $\boldsymbol{log\ 15}$ is understood to be $\boldsymbol{log_{10}15}$

For example: $\boldsymbol{ln\ 15}$ is understood to be $\boldsymbol{log_e 15}$

Both of these logarithms can easily be approximated using a scientific calculator.

To approximate a Common Logarithm, use the calculator's $log$ key.

To approximate a Natural Logarithm, use the calculator's $ln$ key.

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Ex) Use a scientific calculator to approximate each logarithm. Verify each result by evaluating the appropriate exponential expression. Round all answers to three decimal places.

$log(13) \approx$ 1.114

$10^{1.114} \approx 13.002$

$ln(13) \approx$ 2.565

$e^{2.565} \approx 13.001$

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Ex) A biologist studied a population of foxes in a forest preserve over a period of time. From the data collected, the biologist created the model $\overset{\text{dependent}}{t} = 8.85 \ln \frac{\overset{\text{independent}}{P}}{62}$ to predict the amount of time (t) in years it will take for the fox population $P$ to reach a certain number.

a) Describe the domain and range of the function. Include restrictions.

The domain is the fox population when it is greater than or equal to 62.

The range is time in years, where time is non-negative.

b) Estimate how long (t) it will take, in years and months, for the fox population to reach 1000 (P). Round to three decimal places.

① $t = 8.85 \ln \frac{1000}{62} \approx 24.60849\ldots$ years

24 yrs  $\frac{0.60849\cancel{yr}}{1} \cdot \frac{12\text{ mo}}{1\cancel{yr}}$

$\approx 7.302$ mo

② $\boxed{\text{The fox population will reach 1000 after about 24 years 7.302 months.}}$

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Ex) The acidity level, or $pH$, of a liquid is given by the formula $pH = \log \frac{1}{[H^+]}$, where $[H^+]$ is the concentration (in moles per liter) of hydrogen ions in the liquid.

dependent

independent

a) Describe the domain and range of the function. Include restrictions.

The domain is the concentration of hydrogen ions in moles per liter where $0 < [H^+] \leq 1$.

The range is the acidity level of a liquid from 0 to 14.

b) In a typical chlorinated swimming pool, the concentration of hydrogen ions $[H^+]$ ranges from $1.58 \times 10^{-8}$ moles per liter to $6.31 \times 10^{-8}$ moles per liter. What is the range of the $pH$ for a typical swimming pool? Round to the nearest tenth.

1. $pH = \log \frac{1}{(1.58 \times 10^{-8})} \approx 7.8$

   $pH = \log \frac{1}{(6.31 \times 10^{-8})} \approx 7.2$

2. $\boxed{\text{The range of the pH for a typical swimming pool is about 7.2 to 7.8.}}$

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Practice) *Lactobacillus acidophilus* is one of the bacteria used to turn milk into yogurt. The population $P$ of a colony of 3500 bacteria at time $t$ in minutes can be modeled by the function $t = 105.32 \ln \frac{P}{3500}$.

a) Describe the domain and range of the function. Include restrictions.

The domain is the population of lactobacillus acidophilus bacteria greater than or equal to 3500.
The range is the time in minutes greater than or equal to 0.

b) How long, in hours and minutes, does it take the population of bacteria to reach 1,792,000? Round the final answer to the nearest minute.

It will take about 10 hours 57 minutes for the bacteria population to reach 1,792,000.

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Practice) The intensity level $L$, in decibels $dB$, of a sound is given by the formula $L = 10\log\frac{I}{I_0}$ where $I$ is the intensity in watts per square meter, $W/m^2$, of the sound and $I_0$ is the intensity of the softest audible sound, about $10^{-12}\ W/m^2$.

a) Describe the domain and range of the function. Include restrictions.

The domain is the intensity of the sound in watts per square meter, where the intensity is greater than or equal to $10^{-12}$ watts per square meter.
The range is the intensity level of a sound in decibels, where the intensity level is non-negative.

b) What is the intensity level of a rock concert if the sound has an intensity of $3.2\ W/m^2$? Round to the nearest decibel.

The sound of the rock concert has an intensity level of about 125 $dB$.

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### Closure

Jade is taking a chemistry test and has to find the $pH$ of a liquid given that its hydrogen ion concentration is $7.53 \times 10^{-9}$ moles per liter using the formula $pH = \log\frac{1}{[H^+]}$. Her work is shown. She knows her answer doesn't make sense because the $pH$ scale is from 0 to 14, but she runs out of time on the test. Explain her error and find the correct $pH$.

$$pH = \ln\frac{1}{[H^+]}$$

$$pH = \ln\frac{1}{7.53 \times 10^{-9}}$$

$$pH \approx 18.7$$

Jade's error is that the formula uses the common logarithm but she used the natural logarithm. The correct $pH$ for the liquid is about 8.1.