

Objective: Solve equations with rational exponents.

Concept

**An equation with rational exponents is equivalent to a radical equation.**

When solving these equations it is usually easier to leave them in the original form rather than rewrite in radical form.

Examples

$$x^{\frac{2}{3}} + 8x^{\frac{1}{2}} = 0$$

$$9x - x^{\frac{1}{2}} = 0$$

$$x + x^{\frac{1}{2}} - 6 = 0$$

Objective: Solve equations with rational exponents.

Concept

Steps to Solve an Equation with Rational Exponents

1. **Set the equation equal to zero.**
2. **Factor the expression.**
3. **Use the Zero Product Property.**
4. **Solve for  $x$ .** The steps from here will vary depending on the equation. The steps may include raising to an appropriate power, using the square root property, using basic algebra, or some other strategy you have learned.
5. **Check for Extraneous Solutions** and then write the final solution set.

Equation with Rational Exponents can have **extraneous solutions**:

1. Solutions that fail to make the left side and right side of the equation equal.
2. Solutions that are imaginary or create imaginary values when substituted into the original equation.



Objective: Solve equations with rational exponents.

Ex) Solve the equation.

$$x^{\frac{1}{2}} = 3$$

① square both sides

$$(x^{\frac{1}{2}})^2 = (3)^2$$

$$x^{\frac{1}{2} \cdot 2} = 9$$

$$x^1 = 9$$

$$x = 9$$

② check

$$9^{\frac{1}{2}} \stackrel{?}{=} 3$$

$$\sqrt{9} = 3$$

$$3 = 3 \checkmark$$

solution:  $x = 9$

$$x^{\frac{1}{3}} = 3$$

① cube both sides

$$(x^{\frac{1}{3}})^3 = (3)^3$$

$$x^{\frac{1}{3} \cdot 3} = 27$$

$$x^1 = 27$$

$$x = 27$$

② check

$$27^{\frac{1}{3}} \stackrel{?}{=} 3$$

$$\sqrt[3]{27} = 3$$

$$3 = 3 \checkmark$$

solution  
 $x = 27$

Objective: Solve equations with rational exponents.

Practice) Solve the equation.

$$x^{\frac{1}{4}} = 2$$

$$\left(x^{\frac{1}{4}}\right)^4 = (2)^4$$

$$x = 16$$

$$\text{check : } \sqrt[4]{16} = 2 \text{ (yes)}$$

$$\boxed{\text{solution : } x = 16}$$



Objective: Solve equations with rational exponents.

Ex) Solve the equation.

$$x^{\frac{3}{2}} = 8$$

① raise to power of  $\frac{2}{3}$

$$\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = \left(8\right)^{\frac{2}{3}}$$

$$x^{\frac{3}{2} \cdot \frac{2}{3}} = \left(\sqrt[3]{8}\right)^2$$

② simplify

$$x^1 = \left(\sqrt[3]{8}\right)^2$$

$$x = 2^2$$

$$x = 4$$

③ check

$$(4)^{\frac{3}{2}} \stackrel{?}{=} 8$$

$$(\sqrt{4})^3$$

$$2^3$$

$$8 = 8 \checkmark$$

solution  
 $x = 4$

Objective: Solve equations with rational exponents.

Practice) Solve the equation.

$$x^{\frac{4}{3}} = 81$$

$$\left(x^{\frac{4}{3}}\right)^{\frac{3}{4}} = (81)^{\frac{3}{4}}$$

$$x = \left(\sqrt[4]{81}\right)^3$$

$$x = 3^3 \rightarrow x = 27$$

$$\text{check: } 27^{\frac{4}{3}} = \left(\sqrt[3]{27}\right)^4 = 3^4 = 81(\text{yes})$$

$$\boxed{\text{solution: } x = 27}$$





Objective: Solve equations with rational exponents.

Ex) Solve the equation.

$$x^{\frac{2}{3}} + 4x^{\frac{1}{3}} = 0$$

① factor  
get  $f = x^{\frac{1}{3}}$

$$x^{\frac{1}{3}}(x^{\frac{1}{3}} + 4) = 0$$

$x^{\frac{1}{3} + ?} = x^{\frac{2}{3}}$

② zero product property

$$x^{\frac{1}{3}} = 0 \text{ or } x^{\frac{1}{3}} + 4 = 0$$

$$\frac{-4}{-4} \quad \frac{-4}{-4}$$

$$x^{\frac{1}{3}} = -4$$

③ solve.

$$(x^{\frac{1}{3}})^3 = (0)^3$$

$$(x^{\frac{1}{3}})^3 = (-4)^3$$

$$x^1 = 0$$

$$x = 0 \checkmark$$

$$x^1 = -64$$

$$x = -64 \checkmark$$

④ check

$$x = 0; \quad (0)^{\frac{2}{3}} + 4(0)^{\frac{1}{3}} \stackrel{?}{=} 0$$

$$\frac{\sqrt[3]{0^2}}{\sqrt[3]{0}} + 4 \cdot \frac{\sqrt[3]{0}}{\sqrt[3]{0}} = 0$$

$$0 + 4 \cdot 0 = 0$$

solutions  
 $x = -64, 0$

$$0 + 0 = 0 \checkmark$$

$$x = -64; \quad (-64)^{\frac{2}{3}} + 4(-64)^{\frac{1}{3}} \stackrel{?}{=} 0$$

$$\frac{(\sqrt[3]{-64})^2}{(\sqrt[3]{-64})} + 4 \cdot \frac{\sqrt[3]{-64}}{\sqrt[3]{-64}}$$

$$(-4)^2 + 4 \cdot (-4)$$

$$16 + -16 = 0 \checkmark$$



Objective: Solve equations with rational exponents.

Practice) Solve the equation.

$$x - 3x^{\frac{1}{2}} = 0$$

$$x^{\frac{1}{2}} \left( x^{\frac{1}{2}} - 3 \right) = 0$$

$$x^{\frac{1}{2}} = 0 \quad \text{or} \quad x^{\frac{1}{2}} - 3 = 0$$

$$\left( x^{\frac{1}{2}} \right)^2 = (0)^2 \qquad x^{\frac{1}{2}} = 3$$

$$x = 0$$

$$\left( x^{\frac{1}{2}} \right)^2 = (3)^2$$

$$x = 9$$

$$\text{check : } x = 0; \quad 0 - 3\sqrt{0} = 0 \text{ (yes)}$$

$$\text{check : } x = 9; \quad 9 - 3\sqrt{9} = 0 \rightarrow 9 - 9 = 0 \text{ (yes)}$$

$$\text{solutions : } x = 0, x = 9$$



Objective: Solve equations with rational exponents.

Practice) Solve the equation.

$$x^{\frac{7}{3}} - 4x^{\frac{1}{3}} = 0$$

$$x^{\frac{1}{3}} \left( x^{\frac{6}{3}} - 4 \right) = 0$$

$$x^{\frac{1}{3}} (x^2 - 4) = 0$$

$$x^{\frac{1}{3}} = 0 \quad \text{or} \quad x^2 - 4 = 0$$

$$\left( x^{\frac{1}{3}} \right)^3 = (0)^3 \quad x^2 = 4$$

$$x = 0 \quad \sqrt{x^2} = \pm\sqrt{4}$$

$$x = \pm 2$$

$$\text{check : } x = 0; \quad \sqrt[3]{0^7} - 4\sqrt[3]{0} = 0 \text{ (yes)}$$

$$\text{check : } x = 2; \quad \sqrt[3]{2^7} - 4\sqrt[3]{2} = 0 \rightarrow \sqrt[3]{2^6} \cdot \sqrt[3]{2} - 4\sqrt[3]{2} = 0 \rightarrow \sqrt[3]{(2^2)^3} \cdot \sqrt[3]{2} - 4\sqrt[3]{2} = 0 \rightarrow 4\sqrt[3]{2} - 4\sqrt[3]{2} = 0 \text{ (yes)}$$

$$\text{check : } x = -2; \quad \sqrt[3]{(-2)^7} - 4\sqrt[3]{-2} = 0 \rightarrow \sqrt[3]{(-2)^6} \cdot \sqrt[3]{-2} - 4\sqrt[3]{-2} = 0 \rightarrow \sqrt[3]{((-2)^2)^3} \cdot \sqrt[3]{-2} - 4\sqrt[3]{-2} = 0 \rightarrow 4\sqrt[3]{-2} - 4\sqrt[3]{-2} = 0 \text{ (yes)}$$

$$\text{solutions : } x = -2, x = 0, x = 2$$



Objective: Solve equations with rational exponents.

Ex) Solve the equation.

$$x - 5x^{\frac{1}{2}} - 6 = 0$$

same as:  $x - 5\sqrt{x} - 6 = 0$

①  $x - 5x^{\frac{1}{2}} - 6 = 0$   
 Factor  
 $(x^{\frac{1}{2}} - 6)(x^{\frac{1}{2}} + 1) = 0$

② Z.P.P.  $x^{\frac{1}{2}} - 6 = 0$  or  $x^{\frac{1}{2}} + 1 = 0$

③  $\begin{array}{l} +6 \quad +6 \\ x^{\frac{1}{2}} = 6 \\ (x^{\frac{1}{2}})^2 = (6)^2 \\ x = 36 \checkmark \end{array} \quad \begin{array}{l} -1 \quad -1 \\ x^{\frac{1}{2}} = -1 \\ (x^{\frac{1}{2}})^2 = (-1)^2 \\ x = 1 \\ \text{extraneous} \end{array}$

④ check

$x = 36: 36 - 5\sqrt{36} - 6 \stackrel{?}{=} 0$

$36 - 30 - 6 = 0$   
 $6 - 6 = 0 \checkmark$

$x = 1: 1 - 5\sqrt{1} - 6 \stackrel{?}{=} 0$   
 $1 - 5 - 6 = 0$   
 $-4 - 6$   
 $-10 \neq 0$

①  $x - 5\sqrt{x} - 6 = 0$   
 $(\sqrt{x} - 6)(\sqrt{x} + 1) = 0$

②  $\sqrt{x} - 6 = 0$  or  $\sqrt{x} + 1 = 0$   
 $\begin{array}{l} +6 \quad +6 \\ \sqrt{x} = 6 \\ (\sqrt{x})^2 = (6)^2 \\ x = 36 \checkmark \end{array} \quad \begin{array}{l} -1 \quad -1 \\ \sqrt{x} = -1 \\ (\sqrt{x})^2 = (-1)^2 \\ x = 1 \\ \text{extraneous} \end{array}$

③  $x = 36 \checkmark$   
 $x = 1$   
 extraneous

solution  
 $x = 36$

Objective: Solve equations with rational exponents.

Ex) Solve the equation.

$$x^{\frac{2}{3}} + 4x^{\frac{1}{3}} + 3 = 0$$

① factor  $(x^{\frac{1}{3}} + 3)(x^{\frac{1}{3}} + 1) = 0$

② Z.P.P.  $x^{\frac{1}{3}} + 3 = 0$  or  $x^{\frac{1}{3}} + 1 = 0$

$$\frac{-3}{-3} \qquad \frac{-1}{-1}$$

③  $x^{\frac{1}{3}} = -3$   $x^{\frac{1}{3}} = -1$

$$(x^{\frac{1}{3}})^3 = (-3)^3 \qquad (x^{\frac{1}{3}})^3 = (-1)^3$$

$$x = -27 \checkmark \qquad x = -1 \checkmark$$

④ check.  $x = -27$ ;  $(-27)^{\frac{2}{3}} + 4(-27)^{\frac{1}{3}} + 3 \stackrel{?}{=} 0$

$$\begin{aligned} & (\sqrt[3]{-27})^2 + 4 \cdot \sqrt[3]{-27} + 3 \\ & (-3)^2 + 4 \cdot (-3) + 3 \\ & 9 + (-12) + 3 = 0 \end{aligned}$$

$x = -1$   $(-1)^{\frac{2}{3}} + 4(-1)^{\frac{1}{3}} + 3 \stackrel{?}{=} 0$

$$\begin{aligned} & (\sqrt[3]{-1})^2 + 4 \cdot \sqrt[3]{-1} + 3 \\ & (-1)^2 + 4 \cdot (-1) + 3 \\ & 1 + (-4) + 3 \\ & -3 + 3 = 0 \end{aligned}$$

solutions  
 $x = -27, -1$

Objective: Solve equations with rational exponents.

Practice) Solve the equation.

$$x - 3x^{\frac{1}{2}} + 2 = 0$$

$$\left(x^{\frac{1}{2}} - 2\right)\left(x^{\frac{1}{2}} - 1\right) = 0$$

$$x^{\frac{1}{2}} - 2 = 0 \quad \text{or} \quad x^{\frac{1}{2}} - 1 = 0$$

$$x^{\frac{1}{2}} = 2 \quad \quad \quad x^{\frac{1}{2}} = 1$$

$$\left(x^{\frac{1}{2}}\right)^2 = (2)^2 \quad \quad \quad \left(x^{\frac{1}{2}}\right)^2 = (1)^2$$

$$x = 4 \quad \quad \quad x = 1$$

$$\text{check: } x = 4; 4 - 3\sqrt{4} + 2 = 0 \rightarrow 4 - 6 + 2 = 0 \text{ (yes)}$$

$$x = 1; 1 - 3\sqrt{1} + 2 = 0 \rightarrow 1 - 3 + 2 = 0 \text{ (yes)}$$

$$\text{solutions: } x = 1, x = 4$$



Objective: Solve equations with rational exponents.

Practice) Solve the equation.

$$x^{\frac{2}{3}} - 8x^{\frac{1}{3}} - 20 = 0$$

$$\left(x^{\frac{1}{3}} - 10\right)\left(x^{\frac{1}{3}} + 2\right) = 0$$

$$x^{\frac{1}{3}} - 10 = 0 \quad \text{or} \quad x^{\frac{1}{3}} + 2 = 0$$

$$x^{\frac{1}{3}} = 10 \quad x^{\frac{1}{3}} = -2$$

$$\left(x^{\frac{1}{3}}\right)^3 = (10)^3 \quad \left(x^{\frac{1}{3}}\right)^3 = (-2)^3$$

$$x = 1000 \quad x = -8$$

$$\text{check : } x = 1000; \left(\sqrt[3]{1000}\right)^2 - 8\sqrt[3]{1000} - 20 = 0 \rightarrow 100 - 80 - 20 = 0 \text{ (yes)}$$

$$x = -8; \left(\sqrt[3]{-8}\right)^2 - 8\sqrt[3]{-8} - 20 = 0 \rightarrow 4 + 16 - 20 = 0 \text{ (yes)}$$

$$\boxed{\text{solutions : } x = -8, x = 1000}$$



Objective: Solve equations with rational exponents.

Closure

A student solved an equation with rational exponents. The work is shown. Explain why the solution isn't correct and find the correct solution.

$$x^{\frac{1}{3}} = 8$$

$$\sqrt[3]{x^{\frac{1}{3}}} = \sqrt[3]{8}$$

$$x = 2$$

The solution isn't correct because  $\sqrt[3]{x^{\frac{1}{3}}}$  isn't equal to  $x$ ,  $\sqrt[3]{x^{\frac{1}{3}}} = (x^{\frac{1}{3}})^{\frac{1}{3}} = x^{\frac{1}{9}}$ . The correct solution is 512.

$$x^{\frac{1}{3}} = 8$$

$$\left(x^{\frac{1}{3}}\right)^3 = (8)^3$$

$$x = 512$$