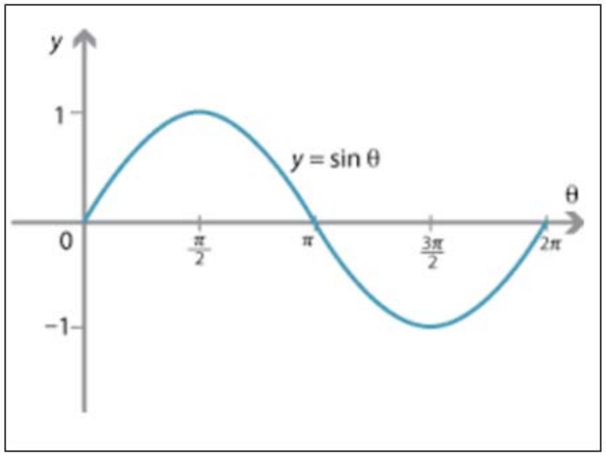
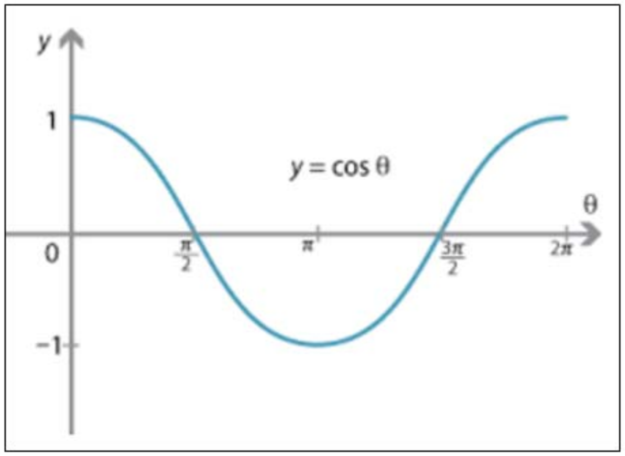
Objective: Solve Trigonometric Equations with more than One Trigonometric Function

Concept

For the function $y = \sin\theta$: the domain, θ, which is all angle measures, is the set of all real numbers and the range, y, is the values of sine in the interval $[-1,1]$.

For the function $y = \cos\theta$: the domain, θ, which is all angle measures, is the set of all real numbers and the range, y, is the values of cosine in the interval $[-1,1]$.





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Steps to Solve a Trigonometric Equation Containing More than One Trigonometric Function

Quadratic Structure

1. Use a Pythagorean Identity to **write the equation in terms of one trigonometric function**.
2. **Use a Quadratic Strategy** (factoring, square root property, quadratic formula) to solve for the trigonometric function values.
3. Find the angle measure(s) that correspond to the function value(s).

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Ex) Solve the equation over the interval $[0, 2\pi)$.

$$2\sin^2 x + 3\cos x - 3 = 0$$

(1) $\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

(2) $2(1 - \cos^2 x) + 3\cos x - 3 = 0$

$2 - 2\cos^2 x + 3\cos x - 3 = 0$

$-1 \cdot [-2\cos^2 x + 3\cos x - 1 = 0]$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$2\cos x - 1 = 0$ or $\cos x - 1 = 0$

$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = 1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\qquad$ $x = 0$ radians

$$\boxed{x = 0, \frac{\pi}{3}, \frac{5\pi}{3}}$$

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Steps to Solve a Trigonometric Equation Containing More than One Trigonometric Function

Linear Structure

1. **Isolate one trigonometric function on each side** of the equation.
2. **Square both sides** to create a quadratic structure.
3. Use a Pythagorean Identity to write the equation in terms of one trigonometric function.
4. Use a Quadratic Strategy (factoring, square root property, quadratic formula) to solve for the trigonometric function values.
5. Find the angle measure(s) that correspond to the function value(s).
6. **Check for extraneous solutions.**

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Ex) Solve each equation over the interval $[0, 2\pi)$.

① $\cos x + 1 = \sin x$

② $(\cos x + 1)^2 = (\sin x)^2$

$(\cos x + 1)(\cos x + 1)$ $\quad$ $\sin x \cdot \sin x$

$\cos^2 x + \cos x + \cos x + 1 = \sin^2 x$

$\cos^2 x + 2\cos x + 1 = \sin^2 x$

③ $\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$1\cos^2 x + 2\cos x + 1 = 1 - \cos^2 x$

$+1\cos^2 x \quad -1 \quad -1 \quad +\cos^2 x$

$2\cos^2 x + 2\cos x = 0$

④ $2\cos x(\cos x + 1) = 0$

$\frac{2\cos x}{2} = \frac{0}{2}$ or $\cos x + 1 = 0$

$\cos x = 0$ $\quad$ $\cos x = -1$

⑤ $x = \frac{\pi}{2}$ ✓, ~~$\frac{3\pi}{2}$~~ $\quad$ $x = \pi$ ✓

⑥ check!

$\cos x + 1 = \sin x$

$\cos \frac{\pi}{2} + 1 \overset{?}{=} \sin \frac{\pi}{2}$

$0 + 1 = 1$

$1 = 1$

$\cos \frac{3\pi}{2} + 1 \overset{?}{=} \sin \frac{3\pi}{2}$

$0 + 1 = -1$

$1 \neq -1$

$\cos \pi + 1 \overset{?}{=} \sin \pi$

$-1 + 1 = 0$

$0 = 0$

solution: $x = \frac{\pi}{2}, \pi$

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Closure

How do you know if a trigonometric equation has linear structure? How do you know if a trigonometric equation has quadratic structure?

A trigonometric equation has linear structure if all trigonometric functions (sin, cos, tan) are to the first power. A trigonometric equation has quadratic structure if the highest power of a trigonometric function is 2.