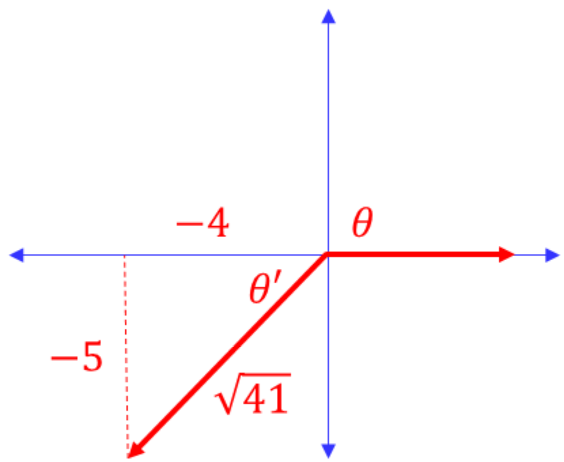
Objective: Find the exact sine or cosine of the sum or difference of two general angle measures.

## Prior Knowledge

If $\tan\theta = \frac{5}{4}$, and $\theta$ terminates in Quadrant III, find $\sin\theta$ .




In QIII, $x$ and $y$ are negative $\rightarrow x=-4,\ y=-5$

$(-4)^2+(-5)^2=r^2$

$r=\sqrt{41}$

$$\boxed{\sin\theta=-\frac{5\sqrt{41}}{41}}$$

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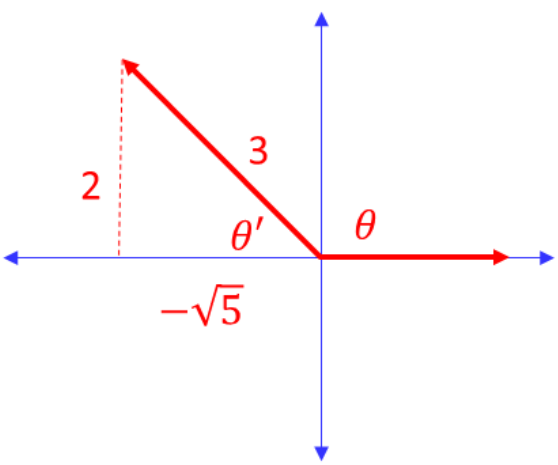
## Prior Knowledge

If $\sin\theta = \frac{2}{3}$, and $\theta$ terminates in Quadrant II, find $\cos\theta$ .




$2^2 + x^2 = 3^2$

$x = \pm\sqrt{3^2 - 2^2} = \pm\sqrt{5}$

In QII, $x$ is negative $\rightarrow x = -\sqrt{5}$

$$\boxed{\cos\theta = -\frac{\sqrt{5}}{3}}$$

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## Concept

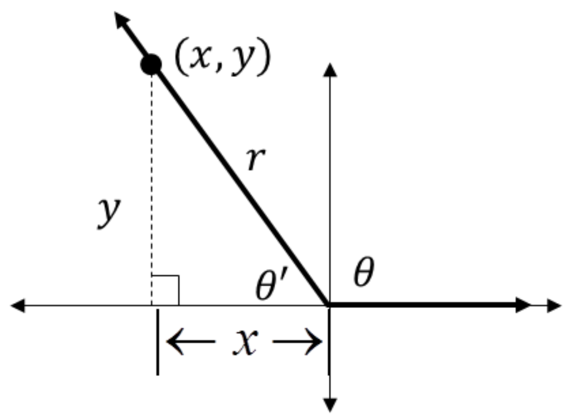
For an angle $\theta$ in standard position with point $(x, y)$ on its terminal side, the six trigonometric functions can be defined by the following:

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

$$where\ x^2 + y^2 = r^2$$



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## Concept

Given two angles, $\alpha$ and $\beta$, in standard position, with known values of at least one trigonometric function for each angle and the quadrant in which each angle terminates, the sum and difference identities can be used to find the exact value for the sine and cosine of the angle that is the sum of the angles and the angle that is the difference of the angles.

| *Sum and Difference Identities for Trigonometric Functions* |
| --- |
| $sin(\alpha+\beta) = sin\,\alpha\cos\beta + \cos\alpha\sin\beta$ |
| $sin(\alpha-\beta) = sin\,\alpha\cos\beta - \cos\alpha\sin\beta$ |
| $cos(\alpha+\beta) = cos\,\alpha\cos\beta - \sin\alpha\sin\beta$ |
| $cos(\alpha-\beta) = cos\,\alpha\cos\beta + \sin\alpha\sin\beta$ |

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**Steps for Using a Sum or Difference Identity**

1. Draw reference triangles for each angle in the appropriate quadrant and use the Pythagorean Theorem to find the missing side.
2. Identify the identity to be used. Substitute the values into the identity using the triangles for reference.
3. Simplify.
4. Write the final answer.

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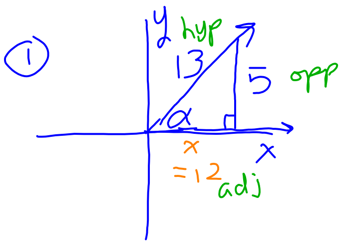
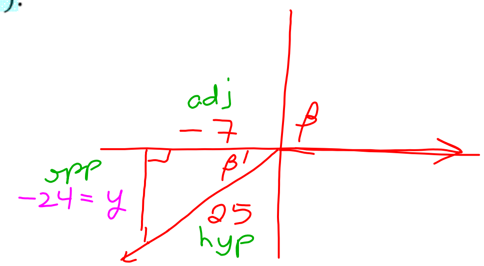
Ex) Given $\sin\alpha = \frac{5\,\text{opp}}{13\,\text{hyp}}$, where $0 < \alpha < \frac{\pi}{2}$ and $\cos\beta = -\frac{7\,\text{adj}}{25\,\text{hyp}}$, where $\pi < \beta < \frac{3\pi}{2}$,
find the exact value of $\sin(\alpha - \beta)$.

① 


$$x^2 + 5^2 = 13^2$$
$$x^2 + 25 = 169$$
$$-25 \quad -25$$
$$x^2 = 144$$
$$x = \pm\sqrt{144}$$
$$= \pm 12 \rightarrow 12$$

$$(-7)^2 + y^2 = 25^2$$
$$49 + y^2 = 625$$
$$-49 \qquad -49$$
$$y^2 = 576$$
$$y = \pm\sqrt{576}$$
$$y = \pm 24 \rightarrow -24$$

② identity: $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

$$\sin(\alpha - \beta) = \frac{5}{13} \cdot \frac{-7}{25} - \frac{-24}{25} \cdot \frac{12}{13}$$

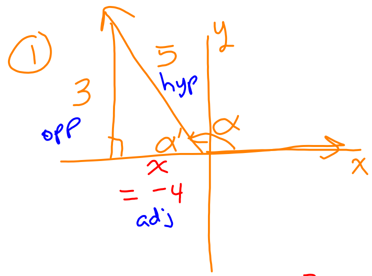
$$= \frac{-35}{325} + \frac{+288}{325}$$

④ $\boxed{\sin(\alpha - \beta) = \frac{253}{325}}$

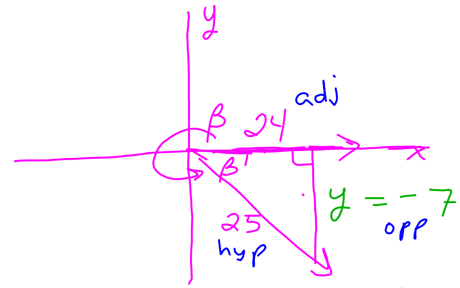
Objective: Find the exact sine or cosine of the sum or difference of two general angle measures.

Ex) Given $\sin\alpha = \frac{3}{5}$, where $\frac{\pi}{2} < \alpha < \pi$ and $\cos\beta = \frac{24}{25}$, where $\frac{3\pi}{2} < \beta < 2\pi$, find the exact value of $\cos(\alpha+\beta)$.

① 




$$x^2 + 3^2 = 5^2$$
$$x^2 + 9 = 25$$
$$x^2 = 16$$
$$x = \pm 4 \to -4$$




$$24^2 + y^2 = 25^2$$
$$576 + y^2 = 625$$
$$y^2 = 49$$
$$y = \pm 7 \to -7$$

② identity: $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

③ $\cos(\alpha+\beta) = \frac{-4}{5}\cdot\frac{24}{25} - \frac{3}{5}\cdot\frac{-7}{25}$

$$\cos(\alpha+\beta) = \frac{-96}{125} + \frac{+21}{125}$$

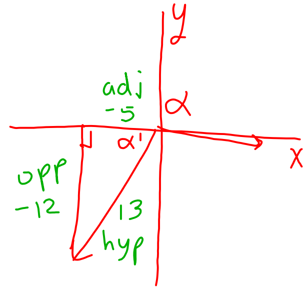
④ $\cos(\alpha+\beta) = \frac{-75}{125}$

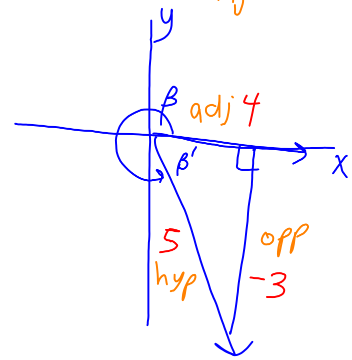
$$\boxed{\cos(\alpha+\beta) = \frac{-3}{5}}$$

Objective: Find the exact sine or cosine of the sum or difference of two general angle measures.

Ex) Given $\tan\alpha = \frac{12}{5}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\tan\beta = -\frac{3}{4}$, where $\frac{3\pi}{2} < \beta < 2\pi$, find the exact value of $\cos(\alpha - \beta)$.

opp adj opp adj

① 


② $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\frac{-5}{13} \cdot \frac{4}{5} + \frac{-12}{13} \cdot \frac{-3}{5}$$

③ $\cos(\alpha - \beta) = \frac{-20}{65} + \frac{36}{65}$

④ $\boxed{\cos(\alpha - \beta) = \frac{16}{65}}$

Objective: Find the exact sine or cosine of the sum or difference of two general angle measures.

## Closure

Given that $\cos\alpha = -\frac{6}{7}$, where $\pi < \alpha < \frac{3\pi}{2}$, how do you determine whether the value in the numerator or denominator is negative?

Since cosine is defined as $\frac{x}{r}$ and $r$ is always positive, the value in the numerator is negative.