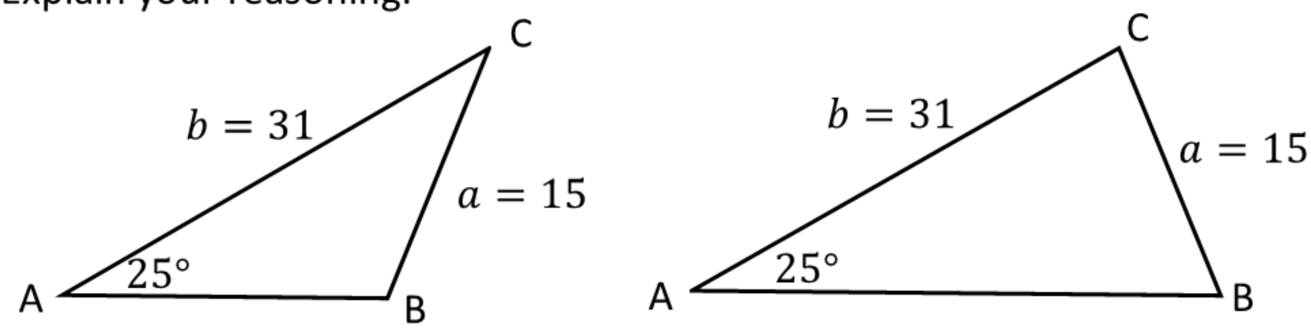
Objective: Solve Triangle Problems Using the Ambiguous Case of the Law of Sines

Concept

Stephanie says these two triangles are congruent by the SSA (side-side-angle) Triangle Congruence Theorem. Do you agree or disagree with Stephanie? Explain your reasoning.



I disagree with Stephanie. The two triangles are not congruent for two reasons. First, there is no SSA triangle congruence theorem, and second, in one triangle angle B is an obtuse angle and in the other triangle angle B is an acute angle so corresponding parts are not congruent.

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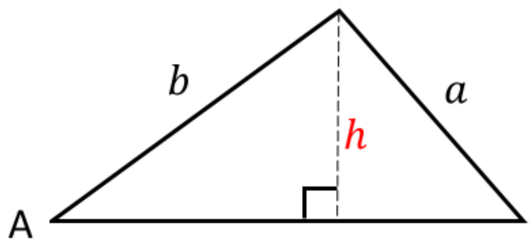
Concept

Solving a Triangle when SSA is Known Information

Since two sides and a non-included angle do not define a unique triangle, having this information creates an ambiguous case involving the Law of Sines. **A test must be done to determine whether the two side measures and non-included angle create no triangle, one triangle, or two triangles.**

Recall that the height of a triangle can be found using the product of the sine of the angle opposite the height and the adjacent side length that is not the base.



$$\sin A = \frac{h}{b} \rightarrow h = b \cdot \sin A$$

In order to create a triangle, the length of side a must be equal to or greater than the height h.

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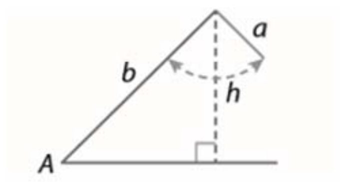
Concept

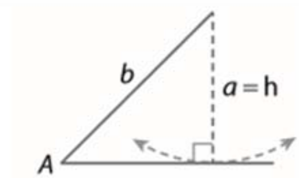
The Ambiguous Case

Given $a, b, and\ m\angle A$

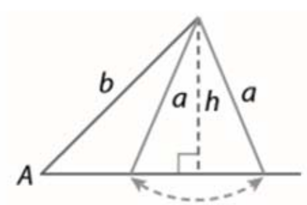
If $\angle A$ is acute.



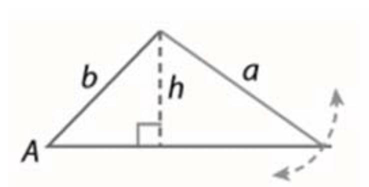
If $a < h$, no triangle exists.



If $a = h$, one triangle exists and it's a right triangle.

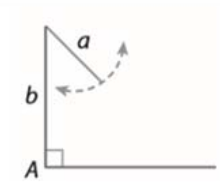


If $h < a < b$, two triangles exist.

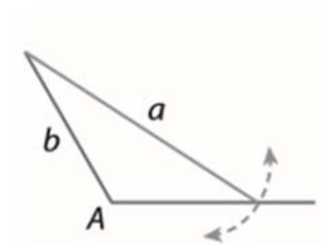


If $a \geq b$, one triangle exists.

If $\angle A$ is right or obtuse.



If angle A is right or obtuse and $a \leq b$, no triangle exists.



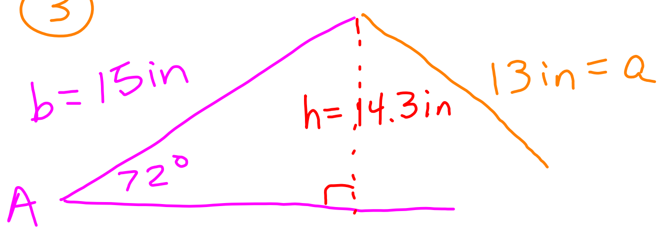
If angle A is right or obtuse and $a > b$, one triangle exists.

Objective: Solve Triangle Problems Using the Ambiguous Case of the Law of Sines

Ex) Given two sides and a non-included angle, determine how many triangles exist.

$$a = 13\ in, b = 15\ in, m\angle A = 72°$$

3



1 find the height

$h = b \cdot \sin A$

$h = 15 \cdot \sin 72°$

$h \approx 14.3\ in$

2 compare h and side a

$14.3\,in > 13\,in$

$h > a$

4 $\boxed{\text{no triangle exists}}$

Objective: Solve Triangle Problems Using the Ambiguous Case of the Law of Sines

Ex) Given two sides and a non-included angle, determine how many triangles exist.

$$a = 10.4\ in, b = 12.3\ in, m\angle A = 49°$$

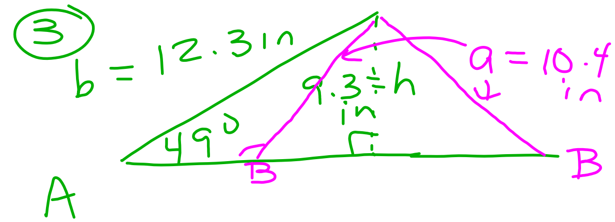
① find the height

$h = b \cdot \sin A$

$h = 12.3 \cdot \sin 49^0$

$h \approx 9.3$ in

③



④ $\boxed{\text{Two triangles exist}}$

② compare h and side a

9.3 in < 10.4 in

h < a

Objective: Solve Triangle Problems Using the Ambiguous Case of the Law of Sines

Ex) Given two sides and a non-included angle, determine how many triangles exist.

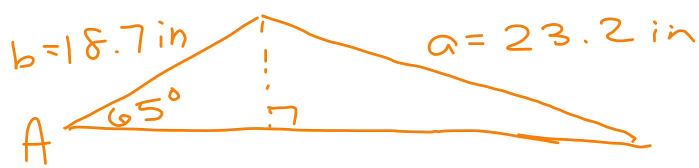
$$a = 23.2\ in, b = 18.7\ in, m\angle A = 65°$$

1 $a > b$

$23.2in > 18.7in$

3 $\boxed{\text{One triangle exists.}}$

2



Objective: Solve Triangle Problems Using the Ambiguous Case of the Law of Sines

Solving the Ambiguous Case of the Law of Sines

1. Calculate the height of the triangle using $h = b \cdot \sin A$

2. Draw a sketch of the triangle that is approximately to scale for the given measurements. Carefully label the measures.

3. If **one triangle exists**, **use the Law of Sines to find the second angle.** Find the other missing measures.

4. If **two triangles exist**, **use the Law of Sines to find the second angle of the first triangle.** **The second angle of the second triangle will be its supplement.** Sketch both triangles and find the remaining missing measures.

5. It's a good idea to **check the validity of your measurements** by **making sure the largest side is opposite the largest angle and the smallest side is opposite the smallest angle**. **Also, you can use the Triangle Inequality Theorem which says the sum of any two sides must to greater than the third to determine if your triangle measurements make sense**.

Objective: Solve Triangle Problems Using the Ambiguous Case of the Law of Sines

Ex) Determine how many triangles exist. If a triangle exists, find all unknown measures of the triangle. When necessary, round side measures to the nearest tenth and angle measures to the nearest degree.

$a = 16.2\ cm,\ b = 18.7\ cm,\ m\angle A = 19°$

① find the height

$h = b \cdot \sin A$

$h = 18.7 \cdot \sin 19°$

$h \approx 6.1$ cm

② compare h and side a

$6.1\ cm < 16.2\ cm$

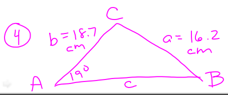
$h < a$

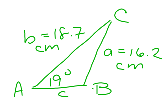
$a > h$

③

Two triangles exist.

④





ⓐ find $m\angle B_{\Delta 1}$

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 19°}{16.2} = \frac{\sin B}{18.7\,cm}$

$16.2 \cdot \sin B = 18.7 \cdot \sin 19°$

$\cancel{\sin^{-1}}(\cancel{\sin} B) = \sin^{-1}\left(\frac{18.7 \cdot \sin 19°}{16.2}\right)$

$B = \sin^{-1}\left(\frac{18.7 \cdot \sin 19°}{16.2}\right)$

$\boxed{m\angle B_{\Delta 1} \approx 22°}$

ⓑ find $m\angle C_{\Delta 1}$

$m\angle A + m\angle B + m\angle C = 180°$

$19° + 22° + m\angle C \approx 180°$

$\boxed{m\angle C_{\Delta 1} \approx 139°}$

ⓒ find side c

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin 19°}{16.2} = \frac{\sin 139°}{c}$

$c \cdot \sin 19° = 16.2 \cdot \sin 139°$

$c = \frac{16.2 \cdot \sin 139°}{\sin 19°}$

$\boxed{c \approx 32.6\ cm}$

ⓐ find $m\angle B_{\Delta 2}$

$m\angle B_{\Delta 1} + m\angle B_{\Delta 2} = 180°$

$22° + m\angle B_{\Delta 2} \approx 180°$

$\boxed{m\angle B_{\Delta 2} \approx 158°}$

ⓑ find $m\angle C_{\Delta 2}$

$19° + 158° + m\angle C_{\Delta 2} \approx 180°$

$\boxed{m\angle C_{\Delta 2} \approx 3°}$

ⓒ find side $c_{\Delta 2}$

$\frac{\sin A}{a} = \frac{\sin C_{\Delta 2}}{c}$

$\frac{\sin 19°}{16.2} = \frac{\sin 3°}{c}$

$c \cdot \sin 19° = 16.2 \cdot \sin 3°$

$c = \frac{16.2 \cdot \sin 3°}{\sin 19°}$

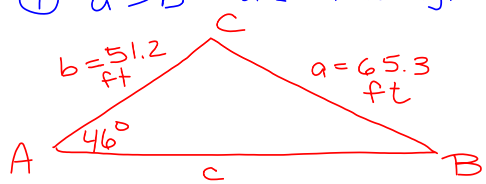
$\boxed{c \approx 2.6\ cm}$

Objective: Solve Triangle Problems Using the Ambiguous Case of the Law of Sines

Ex) Determine how many triangles exist. If a triangle exists, find all unknown measures of the triangle. When necessary, round side measures to the nearest tenth and angle measures to the nearest degree.

$a = 65.3\ ft, b = 51.2\ ft, m\angle A = 46°$

① $a > b$ one triangle exists



② find $m\angle B$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \rightarrow \frac{\sin 46°}{65.3} = \frac{\sin B}{51.2}$$

$$65.3 \cdot \sin B = 51.2 \cdot \sin 46°$$

$$\cancel{\sin^{-1}}\left(\cancel{\sin B}\right) = \sin^{-1}\left(\frac{51.2 \cdot \sin 46°}{65.3}\right)$$

$$B = \sin^{-1}\left(\frac{51.2 \sin 46°}{65.3}\right)$$

$$\boxed{m\angle B \approx 34°}$$

③ find $m\angle C$

$$46° + 34° + m\angle C \approx 180°$$

$$\boxed{m\angle C \approx 100°}$$

④ find side c

$$\frac{\sin 46°}{65.3} = \frac{\sin 100°}{c}$$

$$c = \frac{65.3 \cdot \sin 100°}{\sin 46°}$$

$$\boxed{c \approx 89.4\ ft}$$

Objective: Solve Triangle Problems Using the Ambiguous Case of the Law of Sines

Ex) Determine how many triangles exist. If a triangle exists, find all unknown measures of the triangle. When necessary, round side measures to the nearest tenth and angle measures to the nearest degree.

$$a = 15.9\ in,\ b = 20.3\ in,\ m\angle A = 78°$$

1) find the height

$h = b \cdot \sin A$

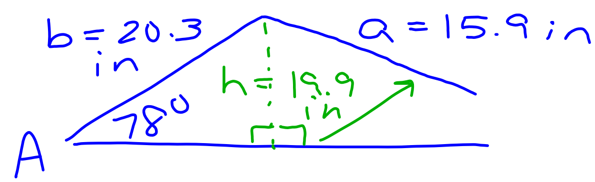
$h = 20.3 \cdot \sin 78°$

$h \approx 19.9\ in$

2) compare h and side a

$19.9\ in > 15.9\ in$

$h > a$



3) $\boxed{\text{no triangle exists}}$

Objective: Solve Triangle Problems Using the Ambiguous Case of the Law of Sines

Closure

When two triangles are possible, how do you find the measure of the second angle of each triangle?

Use the Law of Sines to find the second angle of the first triangle. Subtract this angle from 180° to find the supplement. The supplement will be the measure of the second angle of the second triangle.