Objective: Use Composition to Determine Whether Two Functions are Inverses

<u>Concept</u>

Composition is a mathematical process in which **one function, $f(x)$, uses the range (y values) of another function, $g(x)$, as its domain (x values).** This means the second function (which represents y values) is substituted into the first function for its x values.

The composition of $f(x)$ and $g(x)$.

$$(f \circ g)(x) = f\left(g(x)\right)$$

$$f(x) = x^2 + 2 \quad \textit{and} \quad g(x) = \underbrace{2x+5}_{\textit{range}}$$

domain

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Composition Notation

The composition of two functions, $f(x)$ and $g(x)$, can be written two ways.

1. $(f \circ g)(x)$
2. $f(g(x))$

Both notations can be read "$f\ of\ g\ of\ x$" or "$f\ composition\ g\ of\ x$."

Procedure for the Composition of Functions

1. Substitute the second function into the first function, replacing all variables with the second function.
2. Simplify the expression.
3. Write the new function in standard form and using the composition notation.

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Because the inverse of a function uses the range of the function as its domain, composition can be used to verify two functions are inverse functions.

Given two functions, $f(x)$ and $g(x)$, the functions are inverse functions if the following two conditions are met:

1. $(f \circ g)(x) = f(g(x)) = x$
2. $(g \circ f)(x) = g\big(f(x)\big) = x$

Note: If one of the two conditions isn't met, this is enough to conclude the functions are not inverse functions.

Given $f(x) = 4x + 8$ and $g(x) = \frac{1}{4}x - 2$

1. $f\big(g(x)\big) = f\left(\frac{1}{4}x - 2\right) = 4\left(\frac{1}{4}x - 2\right) + 8 = x - 8 + 8 = x$
2. $g\big(f(x)\big) = g(4x + 8) = \frac{1}{4}(4x + 8) - 2 = x + 2 - 2 = x$
3. Since $f(g(x)) = g\big(f(x)\big) = x$, $f(x)$ and $g(x)$ are inverse functions.

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Ex) Use composition to determine whether each pair of functions are inverse functions.

$f(x) = \frac{1}{3}x + 1$ and $g(x) = 3x - 3$

① Find $(f \circ g)(x) = f(g(x))$

$f(x) = \frac{1}{3}x + 1$ domain

$g(x) = \underbrace{3x - 3}_{\text{range}}$

$f(3x-3) = \underbrace{\frac{1}{3}(3x-3) + 1}_{\text{simplify}}$

$= \frac{1}{3} \cdot 3x - \frac{1}{3} \cdot 3 + 1$

$= x - 1 + 1$

$= x$ ✓

② Find $(g \circ f)(x) = g(f(x))$

$g(x) = 3x - 3$ domain

$f(x) = \underbrace{\frac{1}{3}x + 1}_{\text{range}}$

$g\left(\frac{1}{3}x + 1\right) = \underbrace{3\left(\frac{1}{3}x + 1\right) - 3}_{\text{simplify}}$

$= 3 \cdot \frac{1}{3}x + 3 \cdot 1 - 3$

$= x + 3 - 3$

$= x$ ✓

③ conclusion

Since $(f \circ g)(x) = (g \circ f)(x) = x$, $f(x) = \frac{1}{3}x + 1$ and $g(x) = 3x - 3$ are inverse functions.

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Ex) Use composition to determine whether each pair of functions are inverse functions.

$$f(x) = -2x - 6 \text{ and } g(x) = -\frac{1}{2}x + 3$$

① Find $(f \circ g)(x)$

$f(x) = -2x - 6$ $\qquad$ $g(x) = \underbrace{-\frac{1}{2}x + 3}_{\text{range}}$

domain

$f\left(-\frac{1}{2}x+3\right) = \underbrace{-2\left(-\frac{1}{2}x+3\right) - 6}_{\text{simplify}}$

$= -2 \cdot -\frac{1}{2}x + -2 \cdot 3 - 6$

$= x + -6 - 6$

$= x - 12 \neq x$

② conclusion

$$\boxed{\text{Since } (f \circ g)(x) \neq x \text{, the functions } f(x) = -2x-6 \text{ and } g(x) = -\tfrac{1}{2}x + 3 \text{ are not inverse functions.}}$$

Objective: Use Composition to Determine Whether Two Functions are Inverses

Ex) Use composition to determine whether each pair of functions are inverse functions.

$$f(x) = (x-2)^2 + 3 \text{ and } g(x) = \sqrt{x-3} - 2$$

① $(f \circ g)(x)$

$f(\underbrace{x}) = (\underbrace{x} - 2)^2 + 3$ domain

$g(x) = \underbrace{\sqrt{x-3} - 2}_{\text{range}}$

$f(\sqrt{x-3} - 2) = \underbrace{(\sqrt{x-3} - 2 - 2)^2 + 3}_{\text{simplify}}$

$= (\sqrt{x-3} - 4)^2 + 3$

$(\sqrt{x-3} - 4)(\sqrt{x-3} - 4)$

$(\sqrt{x-3})^2 - 8\sqrt{x-3} + 16$

$= x - 3 - 8\sqrt{x-3} + 16 + 3$

$= x - 8\sqrt{x-3} + 16 \neq x$

② Since $(f \circ g)(x) \neq x$, $f(x) = (x-2)^2 + 3$ and $g(x) = \sqrt{x-3} - 2$ are not inverse functions.

Objective: Use Composition to Determine Whether Two Functions are Inverses

Ex) Use composition to determine whether each pair of functions are inverse functions.

$f(x) = (x+4)^2 - 1$ and $g(x) = \sqrt{x+1} - 4$

① $(f \circ g)(x)$

$f(x) = (x+4)^2 - 1$ $\quad g(x) = \underbrace{\sqrt{x+1} - 4}_{\text{range}}$

domain

$f(\sqrt{x+1} - 4) = \underbrace{(\sqrt{x+1} \cancel{-4} \overset{+0}{\cancel{+4}})^2 - 1}_{\text{simplify}}$

$= (\sqrt{x+1})^2 - 1$

$= x + 1 - 1 = x$ ✓

② $(g \circ f)(x)$

$g(x) = \sqrt{x+1} - 4$ $\quad f(x) = \underbrace{(x+4)^2 - 1}_{\text{range}}$

domain

$g((x+4)^2 - 1) = \sqrt{(x+4)^2 \cancel{-1} \overset{}{\cancel{+1}}} - 4$ $\quad +0$

$= \sqrt{(x+4)^2} - 4$

$= x \overset{+0}{\cancel{+4}} \cancel{-4} = x$ ✓

③ $\boxed{\text{Since } (f \circ g)(x) = (g \circ f)(x) = x, \; f(x) = (x+4)^2 - 1 \text{ and } g(x) = \sqrt{x+1} - 4 \text{ are inverse functions.}}$

Objective: Use Composition to Determine Whether Two Functions are Inverses

Ex) Use composition to determine whether each pair of functions are inverse functions.

$$f(x) = x^3 - 4 \text{ and } g(x) = \sqrt[3]{x+4}$$

① $(f \circ g)(x)$

$f(x) = x^3 - 4$ $\qquad$ $g(x) = \underbrace{\sqrt[3]{x+4}}_{\text{range}}$

domain

$f(\sqrt[3]{x+4}) = (\sqrt[3]{x+4})^3 - 4$

$= x + 4 - 4 = x$ ✓

② $(g \circ f)(x)$

$g(x) = \sqrt[3]{x+4}$ $\qquad$ $f(x) = \underbrace{x^3 - 4}_{\text{range}}$

domain

$g(x^3 - 4) = \sqrt[3]{x^3 \cancel{-4} \cancel{+4}}$ $\;+0$

$= \sqrt[3]{x^3} = x$ ✓

③ $\boxed{\text{Since } (f \circ g)(x) = (g \circ f)(x) = x, \; f(x) = x^3 - 4 \text{ and } g(x) = \sqrt[3]{x+4} \text{ are inverse functions.}}$

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Closure

Sylvia used composition to determine whether the two functions $f(x)$ and $g(x)$ are inverse functions. Her conclusion is shown. Do you agree or disagree with Sylvia and why.

1. $(f \circ g)(x) = x + 3$

2. $(g \circ f)(x) = x + 3$

3. Since $f\big(g(x)\big) = g(f(x))$ the functions $f(x)$ and $g(x)$ are inverse functions.

I disagree with Sylvia. For two functions to be inverse functions, both compositions must be equal to x, not just each other.