Objective: Divide complex numbers

Concept

To Multiply Three Complex Numbers

1. Multiply two complex numbers.
2. Multiply the result by the third complex number.

Ex) Simplify each expression.

$$(9+6i)(4+2i)(3-i)$$

(1)

$9(4+2i)+6i(4+2i)$

$36+18i+24i+12i^2$

$12\cdot -1$

-12

$36+-12+18i+24i$

$(24+42i)(3-i)$ (2)

$24(3-1i)+42i(3-1i)$

$72-24i+126i-42i^2$

$-42\cdot -1$

$+42$

$72+42+-24i+126i$

$114+102i$ (3) a+bi form

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Concept

Conjugates are expressions with two terms in which the first terms are the same and the second terms are opposites.

The conjugate of $3 + 4i$ is $3 - 4i$.

The conjugate of $-2 - 5i$ is $-2 + 5i$.

The conjugate of $3x + 5$ is $3x - 5$.

The conjugate of $-1 - \sqrt{7}$ is $-1 + \sqrt{7}$.

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Concept

How to Divide Two Complex Numbers

1. Multiply the quotient of the complex numbers by a ratio equivalent to 1. Create this ratio using the conjugate of the denominator.
2. Perform the multiplication between the numerators and between the denominators.
3. Write the result in the form $a + bi$, reducing fractions where necessary.

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Ex) Simplify each expression.

$$\frac{10}{-6-2i}$$

① find the conjugate of the denominator

$-6+2i$

② multiply by a ratio of 1

$$\frac{10}{(-6-2i)} \cdot \frac{(-6+2i)}{(-6+2i)} = \frac{10(-6+2i)}{-6(-6+2i)+-2i(-6+2i)}$$

$$= \frac{-60+20i}{36-12i+12i-4i^2} = \frac{-60+20i}{40}$$

$i^2 = -1$, $-4 \cdot -1 = +4$, $-12i + 12i = 0$

③ a+bi form

$$\frac{-60}{40} + \frac{20}{40}i$$

$$= \boxed{\frac{-3}{2} + \frac{1}{2}i}$$

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Ex) Simplify each expression.

$$\frac{2-5i}{5-5i}$$

① conjugate of denominator

* $5+5i$

② $\dfrac{(2-5i)}{(5-5i)} \cdot \dfrac{(5+5i)}{(5+5i)}$

$$\frac{2(5+5i)+-5i(5+5i)}{5(5+5i)+-5i(5+5i)} = \frac{10+10i-25i-25i^2}{25+25i-25i-25i^2}$$

$+25$ $\quad -25 \cdot -1$ $\quad +25$ $\quad -25 \cdot 1$

$$= \frac{35-15i}{50}$$

③ $a+bi$ form

$$\frac{35}{50} - \frac{15}{50}i = \boxed{\frac{7}{10} - \frac{3}{10}i}$$

Objective: Divide complex numbers

Ex) Simplify each expression.

$$\frac{3-2i}{-2+5i}$$

① conjugate of the denominator $-2-5i$

② $\frac{(3-2i)}{(-2+5i)} \cdot \frac{(-2-5i)}{(-2-5i)} = \frac{3(-2-5i)+-2i(-2-5i)}{-2(-2-5i)+5i(-2-5i)}$

$= \frac{-6-15i+4i+10i^2}{4+10i-10i-25i^2} = \frac{-16-11i}{29}$

$10 \cdot -1 = -10$, $-25 \cdot -1 = +25$

③ $a+bi$ form

$$\frac{-16}{29} - \frac{11}{29}i$$

Objective: Divide complex numbers

Ex) Simplify each expression.

$$\frac{6+2i}{3i}$$

① conjugate of the *denominator

* $3i = 0 + 3i$

conjugate $= 0 - 3i$

$= -3i$

② $\frac{(6+2i)}{3i} \cdot \frac{-3i}{-3i}$

$= \frac{-3i(6+2i)}{3i \cdot -3i} = \frac{-18i - 6i^2}{-9i^2}$

$-6 \cdot -1 = 6$

$-9 \cdot -1$

$= \frac{6 - 18i}{9}$

③ a+bi form

$= \frac{6}{9} - \frac{18}{9}i$

$= \boxed{\frac{2}{3} - 2i}$