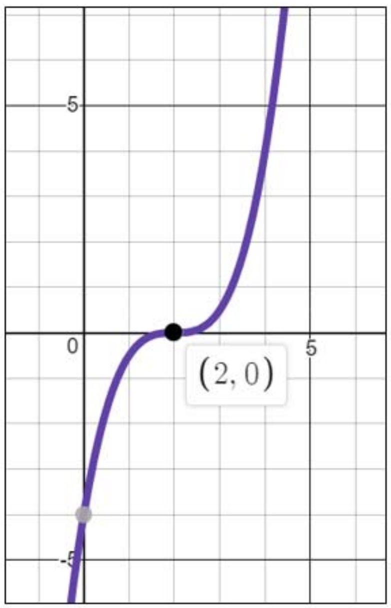
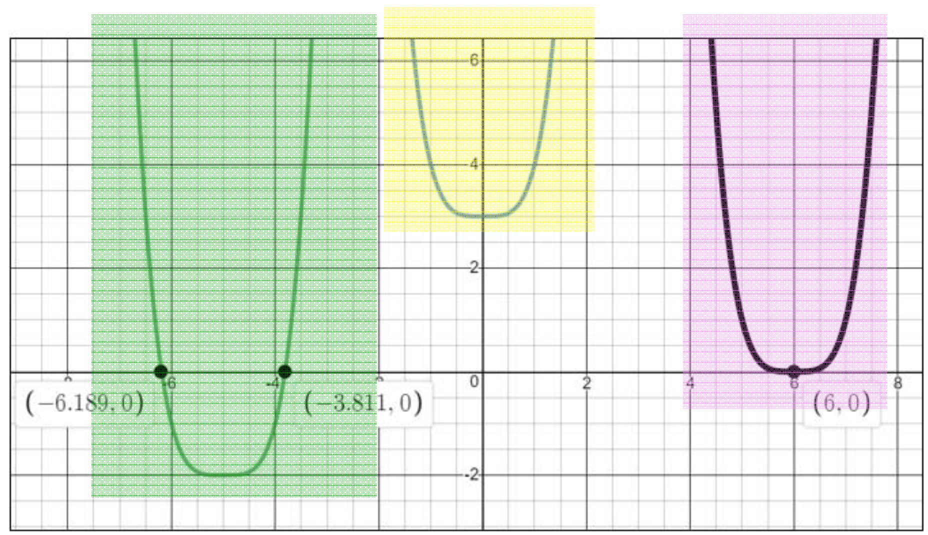
Objective: Find the zeros of cubic and quartic functions algebraically.

## Concept

A **cubic function** of the form $f(x) = a\left(\frac{1}{b}(x-h)\right)^3 + k$ has one real zero.

A **quartic function** of the form $f(x) = a\left(\frac{1}{b}(x-h)\right)^4 + k$ has one real zero, 2 real zeros, or no real zeros.



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## Concept

**To solve an equation you must undo each operation in the reverse of the order of operations.**

1. Undo addition/subtraction outside parentheses.
2. Undo multiplication/division outside parentheses.
3. Undo exponents outside parentheses.
4. Undo operations inside parentheses using the same undoing order as above.

Objective: Find the zeros of cubic and quartic functions algebraically.

Ex) Find the real zeros of the function algebraically. Identify real zeros as rational or irrational.

$$f(x) = 3(x-4)^3 - 6$$

$$y = 3(x-4)^3 - 6$$

concept
zero = ?
then y = 0

$$0 = 3(x-4)^3 - 6$$

$$+6 \qquad\qquad +6$$

$$\frac{6}{3} = \frac{3(x-4)^3}{3}$$

$$2 = (x-4)^3$$

$$\sqrt[3]{2} = \sqrt[3]{(x-4)^3}$$

$$\sqrt[3]{2} = x - 4$$

$$+4 \qquad\qquad +4$$

the zero

$$4 + \sqrt[3]{2} = x$$

$$\boxed{\text{zero} = 4 + \sqrt[3]{2}\text{; irrational}}$$

Objective: Find the zeros of cubic and quartic functions algebraically.

Ex) Find the real zeros of the function algebraically. Identify real zeros as rational or irrational.

$$y = -x^3 - 27$$

$$0 = -x^3 - 27$$

$$+27 \qquad\qquad +27$$

$$\frac{27}{-1} = \frac{-1x^3}{-1}$$

$$-27 = x^3$$

$$\sqrt[3]{-27} = \cancel{\sqrt[3]{\phantom{x}}}x^{\cancel{3}}$$

$$-3 = x$$

$$\boxed{\text{zero} = -3;\ \text{rational}}$$

Objective: Find the zeros of cubic and quartic functions algebraically.

Ex) Find the real zeros of the function algebraically. Identify real zeros as rational or irrational.

$$y = \frac{1}{5}(x+2)^3 - 25$$

$$0 = \frac{1}{5}(x+2)^3 - 25$$

$$+25 \qquad\qquad +25$$

$$5 \cdot 25 = 5 \cdot \frac{1}{5}(x+2)^3$$

$$125 = (x+2)^3$$

$$\sqrt[3]{125} = \sqrt[3]{(x+2)^3}$$

$$5 = x + 2$$

$$-2 \qquad -2$$

$$x = 3$$

$$\boxed{\text{zero} = 3;\ \text{rational}}$$

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Ex) Find the real zeros of the function algebraically. Identify real zeros as rational or irrational.

$$d(x) = (2x-7)^3$$

concept

zeros = ?

then $y = 0$

$$y = (2x-7)^3$$

$$0 = (2x-7)^3$$

$$\sqrt[3]{0} = \cancel{\sqrt[3]{\vphantom{x}}}\,(2x-7)^{\cancel{3}}$$

$$0 = 2x - 7$$

$$+7 \qquad\qquad +7$$

$$\frac{7}{2} = \frac{2x}{2}$$

$$x = 7/2$$

$\boxed{\text{zero} = \frac{7}{2};\ \text{rational}}$

Objective: Find the zeros of cubic and quartic functions algebraically.

Ex) Find the real zeros of the function algebraically. Identify real zeros as rational or irrational.

**Fourth Root Property**
If $x^4 = c$, then $x = \pm\sqrt[4]{c}$

$m(x) = 2(x+1)^4 - 32$

$y = 2(x+1)^4 - 32$

$0 = 2(x+1)^4 - 32$

$+32 \qquad +32$

$\frac{32}{2} = \frac{2(x+1)^4}{2}$

$16 = (x+1)^4$

$\pm\sqrt[4]{16} = \sqrt[4]{(x+1)^{\cancel{4}}}$

$-2, 2 = x + 1$

$-1 \quad -1 \qquad -1$

$-3, 1 = x$

$\boxed{\text{zeros} = -3, 1 \text{ ; rational}}$

Objective: Find the zeros of cubic and quartic functions algebraically.

Ex) Find the real zeros of the function algebraically. Identify real zeros as rational or irrational.

$$y = -\frac{2}{3}x^4 - 10$$

$$0 = -\frac{2}{3}x^4 - 10$$

$$+10 \qquad\qquad +10$$

$$10 = -\frac{2}{3}x^4$$

$$-\frac{3}{\cancel{2}_1}\cdot\frac{\cancel{10}^5}{1} = -\frac{3}{2}\cdot -\frac{2}{3}x^4$$

$$-15 = x^4$$

$$\pm\sqrt[4]{-15} = \sqrt[4]{x^4}$$

not a real number $= x$

There are no real zeros.