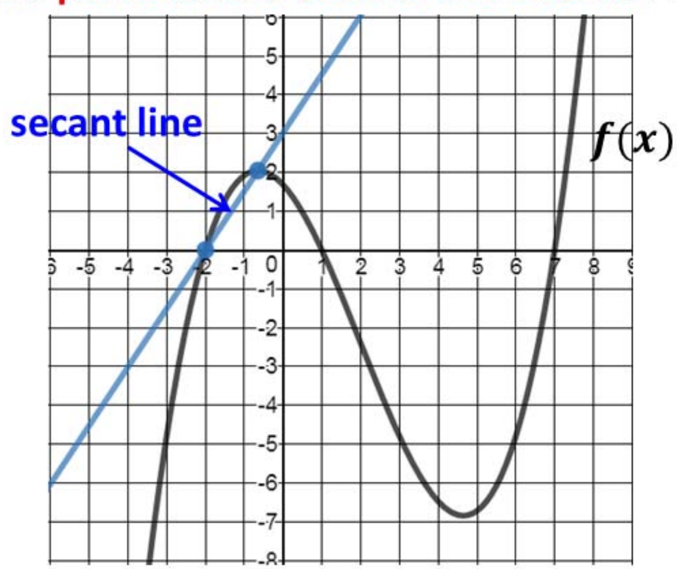
Objective: Calculate and interpret average rate of change in context.

Concept

$$\text{Average Rate of Change } = \frac{\Delta f(x)}{\Delta x}$$

$$\frac{\text{change in } f(x) \text{ values}}{\text{change in } x \text{ values}} = \frac{\Delta f(x)}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \text{ for the interval } [x_1, x_2]$$

- The **average rate of change** is the average **change between y values for each unit of x over a specific interval**.
- The **average rate of change** for an interval **corresponds to the slope of the line through the two points at the ends of the interval**. This line is called the **secant line**.



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Concept

Procedure for Finding an Average Rate of Change

1. Identify the **interval values**.
2. **Find the corresponding function values** for each endpoint of the interval.
3. Use these values in the **average rate of change calculation**. **Include units for the numerator and denominator values.**
4. Interpret in terms of the context. **Be sure to include the units of measure.**

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Ex) The graph shows the height of a butterfly over several minutes as it flies through a park. Approximate the average speed of the butterfly from 7 minutes to 11 minutes. Interpret in terms of the context. Round to three decimal places if necessary.

① interval

$x = 7\text{min}, \; x = 11\text{min}$

② function values

$x = 7\text{min}, \; y = 14\text{ ft}$

$x = 11\text{ min}, \; y = 3.5\text{ft}$

③ AROC

$$\frac{\Delta y}{\Delta x} = \frac{3.5\text{ft} - 14\text{ft}}{11\text{min} - 7\text{min}} = \frac{-10.5\text{ft}}{4\text{ min}}$$

$$= -2.625 \text{ ft per min}$$

$$= -2.625 \text{ ft/min}$$

height is decreasing/descending

avg. speed

④ As the butterfly descends, its average speed from 7 min. to 11 min is about 2.625 ft/min.

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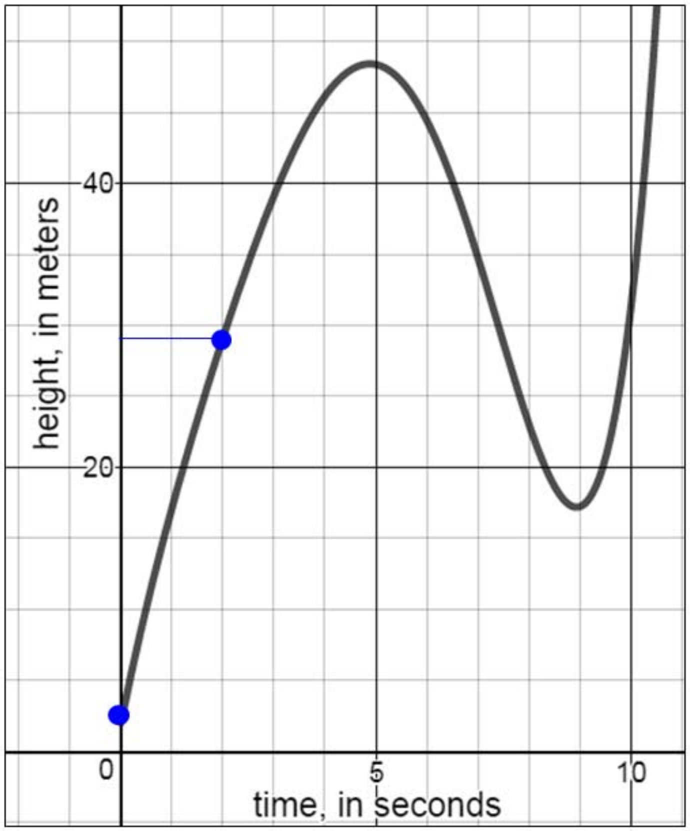
Practice) The graph shows the height of a remote control plane over time. Estimate the the plane's average speed during the first 2 seconds. Interpret in terms of the context. Round to three decimal places if necessary.



1. and 2.

$height \approx 2.5\,meters\ at\ 0\,\text{seconds}$

$height \approx 29\,meters\ at\ 2\,\text{seconds}$

3. $$\frac{\Delta h}{\Delta s} = \frac{29\ meters - 2.5\ meters}{2\,\text{sec} - 0\,\text{sec}}$$

$$= \frac{26.5\,meters}{2\,\text{sec}} = 13.25\,meters/\text{sec}$$

During the first 2 seconds, as the plane ascends, its average speed is about 13.25 meters per second.

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Ex) The function $V(t) = 2t(t-2)^2(t-6)^2 + 5$ represents the value of a company's stock in dollars after t months. Estimate the average change in the value of the stock per month from 1.5 to 4 months.

① interval

$t = 1.5\text{mo}$ $t = 4\text{mo}$

② function value

$t = 1.5\text{mo},\ V(1.5) = 2\cdot1.5(1.5-2)^2\cdot(1.5-6)^2 + 5$

$= \$20.1875$

$t = 4\text{mo},\ V(4) = 2\cdot4(4-2)^2\cdot(4-6)^2 + 5$

$= \$133$

③ AROC

$$\frac{\Delta V}{\Delta t} = \frac{\$133 - \$20.1875}{4\text{mo} - 1.5\text{mo}} = \frac{\$112.8125}{2.5\text{mo}}$$

$= \$45.125$ per month

$= +\$45.13$ per month

↑ stock value is increasing

④ The company's stock value is increasing at an average of about \$45.13 per month from 1.5 to 4 months.

Objective: Calculate and interpret average rate of change in context.

Practice: The function $S(t) = t(t-5)^2(-0.1t-1)^2 + 15$ represents the speed of a car traveling down a curving mountain road in miles per hour and t is the time in minutes since the car entered onto the road. What is the car's average acceleration, in mph/min, from 2 to 4 minutes after entering the road? Round to three decimal places if necessary.

1. find the interval

$t = 2$ minutes to $t = 4$ minutes

2. calculate the function values

$S(2) = 2(2-5)^2(-0.1(2)-1)^2 + 15 = 40.92\,mph$

$S(4) = 4(4-5)^2(-0.1(4)-1)^2 + 15 = 22.84\,mph$

3. calculate the average rate of change

$$\frac{\Delta S}{\Delta t} = \frac{22.84\,mph - 40.92\,mph}{4\min - 2\min}$$

$$= \frac{-18.08\,mph}{2\min} = -9.04\,mph/\min$$

From 2 to 4 minutes after the car enters the road, it is decelerating (slowing down) at an average of 9.04 mph/minute.

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Closure

Ken is doing the average rate of change problem shown. He claims his answer will be in dollars. Why is Ken not correct and what are the correct units for the answer?

The value of a painting in dollars over the years has changed according to the function $V(t) = 0.12(t+2)(t-1)(t-7)+9$. What is the average change per year in the painting's value during the first 5 years?

Ken is not correct because dollars is only the numerator unit in the calculation. The correct units for the answer will be dollars per year.