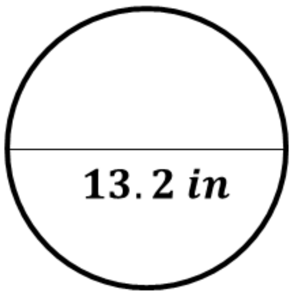
Objective: Find the volume of a cone given context

Prior Knowledge

Find the area of the circle to the nearest tenth.



$$A = \pi r^2$$

$$r = \frac{13.2\,in}{2} = 6.6\,in$$

$$A = \pi \left(6.6\,in\right)^2 = 43.56\pi\,in^2$$

$$A \approx 136.8\,in^2$$

The area of the circle is about 136.8 square inches.

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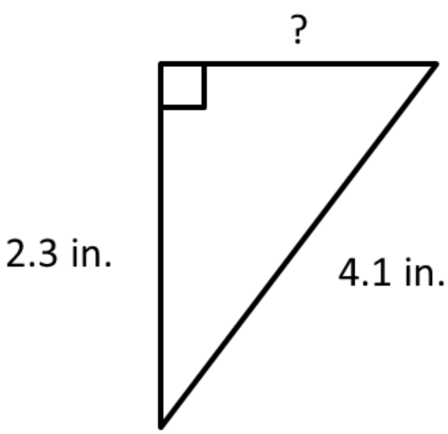
Prior Knowledge

Find the missing side length of the right triangle. Round to the nearest thousandth (three decimal places).



For any right triangle: $a^2 + b^2 = c^2$

So, $(2.3\,in)^2 + b^2 = (4.1\,in)^2$

$$5.29\,in^2 + b^2 = 16.81\,in^2$$

$$b^2 = 11.52\,in^2$$

$$b = \pm\sqrt{11.52\,in^2}$$

$$b = \pm 3.394\,in$$

Since lengths cannot be negative, the side length of the triangle is about 3.394 inches.

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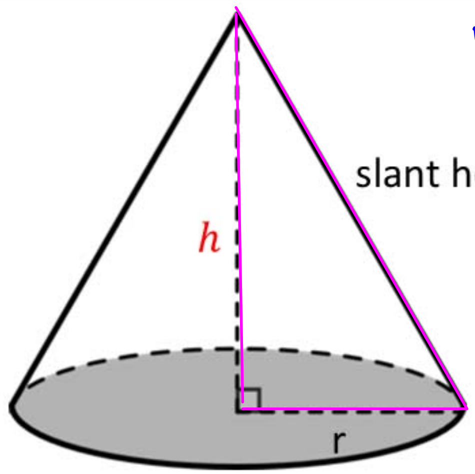
Concept

The formula for the volume V of a cone with base area B is $V = \frac{1}{3}Bh \rightarrow V = \frac{1}{3}(\pi r^2)h$.

Note: The **true height, or height**, h, of a three-dimensional figure is **always the perpendicular distance** from the base of the figure to the highest point. This is always the dimension used for height when calculating volume.

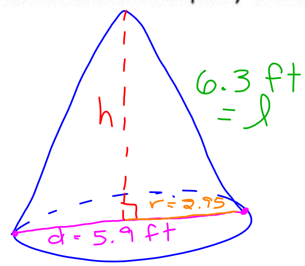
$h^2 + r^2 = \ell^2$



slant height= ℓ

Objective: Find the volume of a cone given context

Ex) A topiary tree is shaped into a cone with a diameter of 5.9 feet and a slant height of 6.3 feet. What is the volume of the topiary tree? Round to the nearest tenth.



$V = \frac{1}{3} B h$

$V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$

① radius

$r = \frac{\text{diameter}}{2} = \frac{5.9\text{ ft}}{2} = 2.95\text{ ft}$

② height * Use Pythagorean Theorem

$a^2 + b^2 = c^2$

$h^2 + r^2 = \ell^2$

$h^2 + 2.95^2 = 6.3^2$

$h^2 + 8.7025 = 39.69$

$-8.7025 \quad -8.7025$

$h^2 = 30.9875$

$\sqrt{h^2} = \sqrt{30.9875}$

$h \approx 5.6\text{ ft}$

③ Volume

$V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$

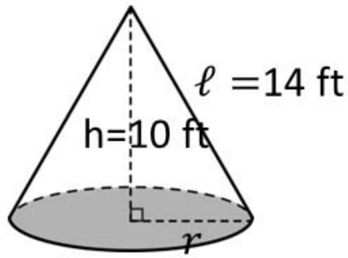
$V = \frac{1}{3} \cdot \pi \cdot (2.95\text{ ft})^2 \cdot 5.6\text{ ft}$

$\approx 51.0\text{ ft}^3$

④ The volume of the topiary tree is about 51.0 cubic feet.

Objective: Find the volume of a cone given context

Practice) The hut is roughly shaped like a cone with a height of 10 feet and a slant height of 14 feet. What is the volume of the hut? Round to the nearest tenth.



1. Find the radius of the base of the hut.

$$(10\,f)t^2 + r^2 = 14\,ft^2$$

$$r^2 = 196\,ft^2 - 100\,ft^2 = 96\,ft^2$$

$$r = \pm\sqrt{96\,ft^2}$$

$$r \approx 9.8\,ft$$

2. Find the volume of the hut.

$$V = \frac{1}{3}Bh$$

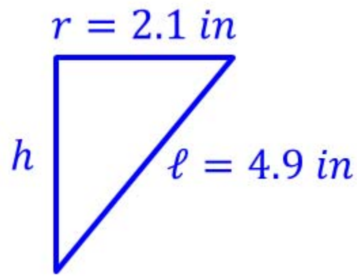
$$V = \frac{1}{3}\cdot \pi \cdot (9.8\,ft)^2\,(10\,ft)$$

$$V \approx 1005.7\,ft^3$$

The volume of the hut is about 1005.7 cubic feet.

Objective: Find the volume of a cone given context

Practice) The glass is roughly cone shaped. Estimate the number of fluid ounces the glass will hold to the nearest tenth. Note: $1\ fl\ oz \approx 1.805\ in^3$



1. Find the height of the glass.

$$(2.1in)^2 + h^2 = (4.9in)^2$$

$$4.41in^2 + h^2 = 24.01in^2$$

$$h = \pm\sqrt{19.6in^2}$$

$$h \approx 4.4in$$

2. Find the volume in cubic inches.

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3}\left(\pi(2.1in)^2\right)\cdot(4.4in)$$

$$V \approx 20.3in^3$$

3. Convert cubic inches to ounces.

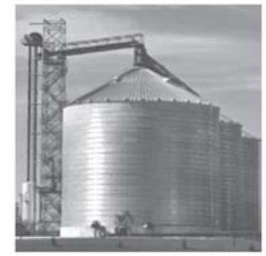
$$20.3in^3 \cdot \frac{1\ fluid\ ounce}{1.805in^3} \approx 11.2\ fluid\ ounces$$

The glass will hold about 11.2 fluid ounces.

Objective: Find the volume of a cone given context

Ex: The roof of a grain silo is in the shape of a cone. The inside radius is 20 feet, and the roof is 10 feet tall. Below the cone is a cylinder 30 feet tall, with the same radius.

a. What is the volume of the silo to the nearest tenth?



① V_{cone} (roof) + $V_{cylinder}$ (body)

$\frac{1}{3} \cdot Bh + B \cdot h$

$\frac{1}{3}\pi r^2 h + \pi r^2 h$

$\frac{1}{3}\pi(20ft)^2 \cdot 10ft + \pi(20ft)^2 \cdot 30ft$

$\approx 41{,}887.9\ ft^3$

② The volume of the grain silo is about 41,887.9 cubic feet.

b. The farmer's crop consists of approximately 2 million pounds of wheat. If one cubic foot of wheat is approximately 48 pounds, will all of the wheat fit in the silo? Explain your reasoning.

$1\ ft^3 \approx 48$ pounds

① convert volume to pounds

$\frac{41{,}887.9\ ft^3}{1} \cdot \frac{48 \text{ pounds}}{1\ ft^3}$

$\approx 2{,}010{,}619.2$ pounds

② All of the wheat will fit in the silo because the silo holds 2,010,619.2 pounds which is more than 2 million pounds.

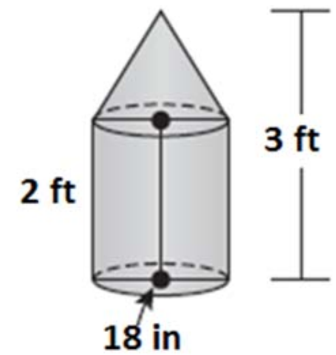
Objective: Find the volume of a cone given context

Practice) Some concrete posts are formed of a cylinder with a cone on top.
a) What is the volume of a post in cubic feet? Round to the nearest tenth.

$$V_{cylinder} = \pi \cdot (0.75\ ft)^2 \cdot (2\ ft) \approx 3.5\ ft^3$$

$$V_{cone} = \frac{1}{3} \cdot \pi \cdot (0.75\ ft)^2 \cdot 1\ ft \approx 0.6\ ft^3$$

$$V_{post} = 3.5\ ft^3 + 0.6\ ft^3 = 4.1\ ft^3$$



The volume of the post is about 4.1 cubic feet.

b) How many pounds of concrete are needed to create 6 posts? Round to the nearest tenth. Note: $1\ ft^3$ of concrete $\approx 150.23\ pounds$.

Pounds of concrete for 1 post:

$$4.1\ ft^3 \cdot \frac{150.23\ pounds}{1\ ft^3} = 615.943\ pounds$$

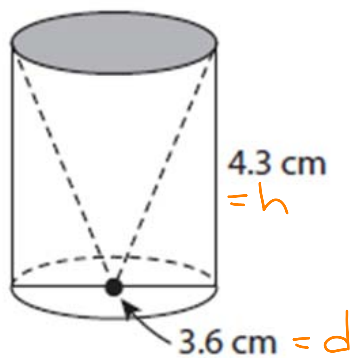
Pounds of concrete for 6 posts:

$$615.943\ pounds \cdot 6\ posts = 3695.7\ pounds$$

To create 6 posts, about 3695.7 pounds of concrete are needed.

Objective: Find the volume of a cone given context

Ex) Karen removed a cone shape from a metal cylinder with the apex (tip) of the cone at the base of the cylinder. Find the volume of the remaining metal. Round to the nearest tenth.



① $r = \frac{3.6\text{cm}}{2}$

$r = 1.8\text{cm}$

② $V_{cylinder} - V_{cone}$

$B \cdot h - \frac{1}{3} \cdot B \cdot h$

$\pi \cdot r^2 \cdot h - \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$

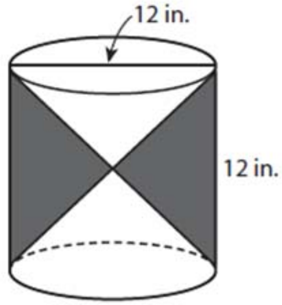
$\pi \cdot (1.8\text{cm})^2 \cdot 4.3\text{cm} - \frac{1}{3} \cdot \pi \cdot (1.8\text{cm})^2 \cdot 4.3\text{cm}$

$\approx 29.2\text{ cm}^3$

③ The volume of the remaining metal is about 29.2 cm^3.

Objective: Find the volume of a cone given context

Practice) Roland is using a special machine to cut cones out of cylindrical pieces of wood. The machine is set to cut out two congruent cones from each piece of wood, leaving no gap in between the vertices of the cones. Find the volume, to the nearest hundredth, of material left over after two cones are cut out?



$$
\begin{aligned}
V_{cylinder} &= Bh \\
&= \pi r^2 \cdot h \\
&= \pi(6\ in)^2 \cdot 12\ in \\
&= 432\pi\ in^3 \\
&\approx 1357.168\ in^3
\end{aligned}
$$

$$
\begin{aligned}
V_{cone} &= \frac{1}{3}Bh \\
&= \frac{1}{3} \cdot \pi r^2 \cdot h \\
&= \frac{1}{3} \cdot \pi(6\ in)^2 \cdot 6\ in \\
&= 72\pi\ in^3 \\
&\approx 226.194\ in^3
\end{aligned}
$$

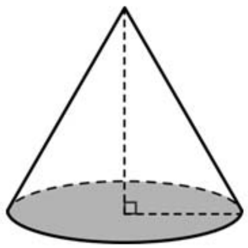
$$
\begin{aligned}
&Volume\ of\ Composite \\
&= Volume\ of\ Cylinder - Volume\ of\ Cone - Volume\ of\ Cone \\
&\approx 1357.168 in^3 - 226.194\ in^3 - 226.194\ in^3 \\
&\approx 904.78\ in^3
\end{aligned}
$$

The volume of material left over after two cones are cut out is about 904.78 cubic inches.

Objective: Find the volume of a cone given context

Closure

The volume for both a cone and square pyramid are $V = \frac{1}{3}Bh$, where B is the base area. Explain how the base area B is different for a cone and square pyramid.



The base area B of a cone is πr^2 while the base area B of a square pyramid is s^2.