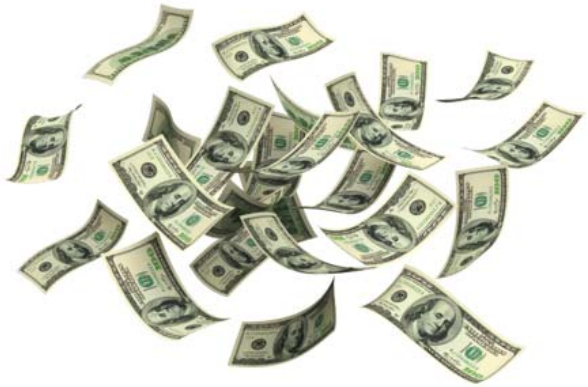
Objective: Solve Problems Involving Compound Interest

Relevance

A *bond* is a type of investment that you buy with cash and for which you receive payment, called interest.

When **interest is paid to you at specific time periods** as the bond matures, it is called **simple interest**.

When **interest is reinvested and also earns interest,** it is called **compound interest**.



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Concept

For $1000 you can buy a bond that matures after 4 years and pays 5% annual interest that is paid to the investor (simple interest).

Simple Interest Bond			
Year	Value of Investment at Beginning of the Year	Interest Earned for Year (Paid to Investor)	Value of Investment at End of Year
1	$1000	$1000(0.05)=$50	$1000
2	$1000	$1000(0.05)=$50	$1000
3	$1000	$1000(0.05)=$50	$1000
4	$1000	$1000(0.05)=$50	$1000

The money earned in interest at the end of the 4 years is $200.

The function, $V(t)$, that represents the value of the investment at the end of a year, t, is the constant function $V(t) = 1000$.

Objective: Solve Problems Involving Compound Interest

Concept

For $1000 you can buy a bond that matures after 4 years and pays 5% annual interest that is reinvested (compound interest).

Compound Interest Bond			
Year	Value of Investment at Beginning of the Year	Interest Earned for Year (Reinvested)	Value of Investment at End of Year
1	$1000	$1000(0.05)=$50	$1050
2	$1050	$1050(0.05)=$52.50	$1102.50
3	$1102.50	$1102.50(0.05)=$55.13	$1157.63
4	$1157.63	$1157.63(0.05)=$57.88	$1215.51

The money earned in interest at the end of the 4 years is $215.51 .

The function, $V(t)$, that represents the value of the investment at the end of a year, t, is the exponential growth function $V(t) = 1000(1.05)^t$.

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Concept

The value of an investment V can be calculated using the function $V(t) = P\left(1 + \frac{r}{n}\right)^{nt}$, where

P is the principle (one-time deposit),

r is the annual interest rate as a decimal,

n is the number of times interest will be compounded (calculated and added to the account balance) in a year,

and t is the time of the investment in years.

Some common compounding periods:

Compounded annually: once a year, $n = 1$

Compounded semiannually (every 6 months), $n = 2$

Compounded quarterly (every 3 months), $n = 4$

Compounded monthly, $n = 12$

Compounded daily, $n = 365$

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Ex) A person invests $3500 ($P$) in an account that earns 3% interest (r) compounded annually. ⇒ once ⇒ $n = 1$

a) Write the function that models this situation.

model: $V(t) = P\left(1 + \frac{r}{n}\right)^{nt}$

$P = 3500$ $\quad r = 3\% = 0.03$ $\quad n = 1$

$V(t) = 3500\left(1 + \frac{.03}{1}\right)^{1 \cdot t}$

$V(t) = 3500(1.03)^{t}$

b) Determine the value of the account after 5 years and 3 months.

$V(t) = ?$ when $t = 5\text{ yr } 3\text{mo}$

$5 + \frac{3\text{mo}}{1} \cdot \frac{1\text{yr}}{12\text{mo}} = 5.25\text{ yr}$

$V = 3500(1.03)^{5.25} = \$4087.55$ →

The value of the account after 5 years and 3 months is $4087.55.

c) How much is earned in interest over the 5 years and 3 months?

$\$4087.55 - \$3500 = \$587.55$

The interest earned over the 5 years and 3 months is $587.55.

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Ex) A person invests $1200 in an account that earns 2.5% annual interest compounded quarterly. → 4 times → $n = 4$

a) Write the function that models this situation.

model

$$V(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

$P = 1200$ $r = 2.5\% = 0.025$ $n = 4$

$$V(t) = 1200\left(1 + \frac{.025}{4}\right)^{4t}$$

$$V(t) = 1200(1.00625)^{4t}$$

b) Find the value of the investment after 11 years and 7 months.

$V(t) = ?$ when $t = 11$ yr 7 mo

$$\frac{7 \text{ mo}}{1} \cdot \frac{1 \text{ yr}}{12 \text{ mo}} = \frac{7}{12}$$

$$= 11\frac{7}{12} \text{ yr} = \frac{139}{12} \text{ yr}$$

$$V = 1200(1.00625)^{4(139/12)}$$

$$= \$1601.60$$

The value of the investment after 11 years and 7 months is $1601.60.

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Concept

The value of an investment V when **interest is compounded continuously** instead of a finite number of times per year can be calculated using $V(t) = Pe^{rt}$, where

P is the principle (one-time deposit),

r is the annual interest rate as a decimal,

and t is the time of the investment in years.

Objective: Solve Problems Involving Compound Interest

Ex) A person invests $5000 (P) in an account that earns 3.5% annual interest (r) compounded continuously.

a) Write the function that models this situation.

model $V(t) = P \cdot e^{rt}$

$P = 5000$

$r = 3.5\% = 0.035$

$V(t) = 5000e^{.035t}$

b) Find the value of the investment after 10 years and 5 months.

$V(t) = ?$ when $t = 10\text{ yr } 5\text{ mo}$

$5\text{ mo} \cdot \frac{1\text{ yr}}{12\text{ mo}}$

$= 10\frac{5}{12}\text{ yr}$

$= \frac{125}{12}\text{ yr}$

$V = 5000 \cdot e^{.035\left(\frac{125}{12}\right)}$

calc.

5000 · 2nd Ln e^(.035 · 125 ÷ 12) =

$= \$7199.57$

The value of the investment after 10 years and 5 months is $7199.57.