Objective: Use the double angle identities to find trigonometric values.

Concept

In the previous lesson, we used the Sum and Difference Identities. The **Double-Angle Identities** are a special case of the Sum Identities, where $\alpha = \beta$.

Here we derive the **Double-Angle Identity for sine**:

$$sin(\alpha + \beta) = sin\alpha \ cos \ \beta + cos\alpha \ sin \ \beta$$

If we let $\beta = \alpha$, then we have

$$sin(\alpha + \alpha) = sin\alpha \ cos\alpha + cos\alpha \ \sin\alpha$$
$$sin(\alpha + \alpha) = sin\alpha \ cos\alpha + sin\alpha \ cos\alpha$$
$$\mathbf{sin(2\alpha) = 2sin\alpha \ cos\alpha}$$

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Concept

The **Double-Angle Identity for cosine** can be derived in a similar way:

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

If we let $\beta = \alpha$, then we have

$$\cos(\alpha+\alpha) = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$$
$$\boldsymbol{\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha}$$

This identity also has useful alternate forms. By substituting $\cos^2\alpha$ with $1-\sin^2\alpha$ from the Pythagorean Identity we obtain:

$$\cos(2\alpha) = 1-\sin^2\alpha - \sin^2\alpha$$
$$\boldsymbol{\cos(2\alpha) = 1 - 2\sin^2\alpha}$$

And, by substituting $\sin^2\alpha$ with $1-\cos^2\alpha$ we obtain:

$$\cos(2\alpha) = \cos^2\alpha - (1-\cos^2\alpha)$$
$$\boldsymbol{\cos(2\alpha) = 2\cos^2\alpha - 1}$$

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Concept

A **Double-Angle Identity for tangent** can also be derived by starting with the Quotient Identity for tangent.

$$\tan(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{\sin\alpha\cos\beta+\sin\beta\cos\alpha}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$$

Letting $\beta=\alpha$, results in the following:

$$\tan(\alpha+\alpha)=\frac{\sin(\alpha+\alpha)}{\cos(\alpha+\alpha)}=\frac{\sin\alpha\cos\alpha+\sin\alpha\cos\alpha}{\cos\alpha\cos\alpha-\sin\alpha\sin\alpha}$$

$$=\frac{2\sin\alpha\cos\alpha}{\cos^2\alpha-\sin^2\alpha}$$

Then, dividing numerator and denominator by $\cos^2\alpha$ gives:

$$\tan(2\alpha)=\frac{\dfrac{2\sin\alpha\cos\alpha}{\cos^2\alpha}}{\dfrac{\cos^2\alpha-\sin^2\alpha}{\cos^2\alpha}}\rightarrow \boldsymbol{\tan(2\alpha)=\frac{2\tan\alpha}{1-\tan^2\alpha}}$$

Objective: Use the double angle identities to find trigonometric values.

Concept

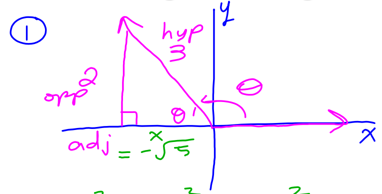
Double-Angle Identities	
$\boldsymbol{sin\,2\alpha = 2\,sin\,\alpha\,cos\alpha}$	$\boldsymbol{cos2\alpha = cos^2\alpha - sin^2\alpha}$ $\boldsymbol{cos2\alpha = 1 - 2sin^2\alpha}$ $\boldsymbol{cos2\alpha = 2cos^2\alpha - 1}$
$\boldsymbol{tan\,2\alpha = \dfrac{2\,tan\,\alpha}{1 - tan^2\alpha}}$	

Objective: Use the double angle identities to find trigonometric values.

Ex) Given $sin\,\theta = \frac{\overset{opp}{2}}{\underset{hyp}{3}}$, where $\frac{\pi}{2} < \theta < \pi$, find each exact value using Double-Angle Identities.

a. $sin2\theta$

①



② $x^2 + y^2 = r^2$

$x^2 + 2^2 = 3^2$

$x^2 + 4 = 9$

$x^2 = 5 \rightarrow x = \pm\sqrt{5}$

$x = -\sqrt{5}$

③ $\sin 2\theta = 2 \cdot \sin\theta \cdot \cos\theta$

$= 2 \cdot \frac{2}{3} \cdot \frac{-\sqrt{5}}{3}$

$\boxed{\sin 2\theta = \frac{-4\sqrt{5}}{9}}$

b. $cos2\theta$, using $\cos 2\theta = 1 - 2\sin^2\theta$

④ $\cos 2\theta = 1 - 2\sin^2\theta$

$= 1 - 2(\sin\theta)^2$

$= 1 - 2\left(\frac{2}{3}\right)^2$

$= 1 - 2\left(\frac{4}{9}\right)$

$= 1 - \frac{8}{9} = \frac{9}{9} - \frac{8}{9}$

$\boxed{\cos 2\theta = \frac{1}{9}}$

c. $tan2\theta$

⑤ $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

$= \frac{2\left(\frac{2}{-\sqrt{5}}\right)}{1 - \left(\frac{2}{-\sqrt{5}}\right)^2}$

$= \frac{\frac{-4}{\sqrt{5}}}{1 - \frac{4}{5}}$

$= \frac{\frac{-4}{\sqrt{5}}}{\frac{1}{5}} = \frac{-4}{\sqrt{5}} \cdot \frac{5}{1}$

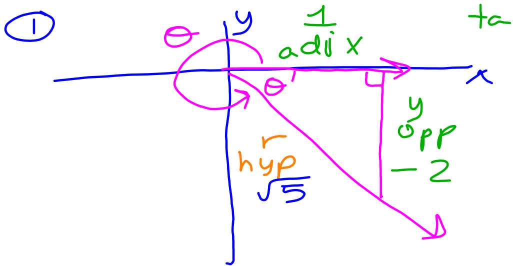
$= \frac{-20}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{-20\sqrt{5}}{5}$

$\boxed{\tan 2\theta = -4\sqrt{5}}$

Objective: Use the double angle identities to find trigonometric values.

Ex) Given $\tan\theta = -2$, where $\frac{3\pi}{2} < \theta < 2\pi$, find each exact value using Double-Angle Identities.

①



$\tan\theta = -\frac{2}{1}$

② $x^2 + y^2 = r^2$

$1^2 + (-2)^2 = r^2$

$5 = r^2$

$r = \sqrt{5}$

a. $sin2\theta$

③ $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{-2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{5}}\right)$

$\boxed{\sin 2\theta = \frac{-4}{5}}$

b. $cos2\theta$, using $\cos 2\theta = cos^2\overset{\theta}{\cancel{\alpha}} - sin^2\overset{\theta}{\cancel{\alpha}}$

④ $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= (\cos\theta)^2 - (\sin\theta)^2$

$= \left(\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{-2}{\sqrt{5}}\right)^2$

$= \frac{1}{5} - \frac{4}{5}$

$\boxed{\cos 2\theta = -\frac{3}{5}}$

c. $tan2\theta$

⑤ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\tan 2\theta = \frac{2(-2)}{1-(-2)^2}$

$= \frac{-4}{1-4}$

$= \frac{-4}{-3}$

$\boxed{\tan 2\theta = \frac{4}{3}}$

Objective: Use the double angle identities to find trigonometric values.

Closure

True or False: $sin2x = 2sinx$. If false, explain why.

False, $sin\,2x = 2\,sin\,x\,cosx$.