Objective: Find the inverse of an exponential function

Concept

The **inverse of a function or relation** is the set of ordered pairs (b, a) obtained by interchanging (switching) the coordinates of each point (a, b) in the original relation or function.

For inverse functions, if $f(x)$ is the original function, then $f^{-1}(x)$ is the inverse function. $f^{-1}(x)$ is read "the inverse of $f(x)$" or "$f\ inverse\ of\ x$".

Because x and y coordinates are interchanged to create the inverse of a function, **the domain of the function becomes the range of the inverse and the range of the function becomes the domain of the inverse**.

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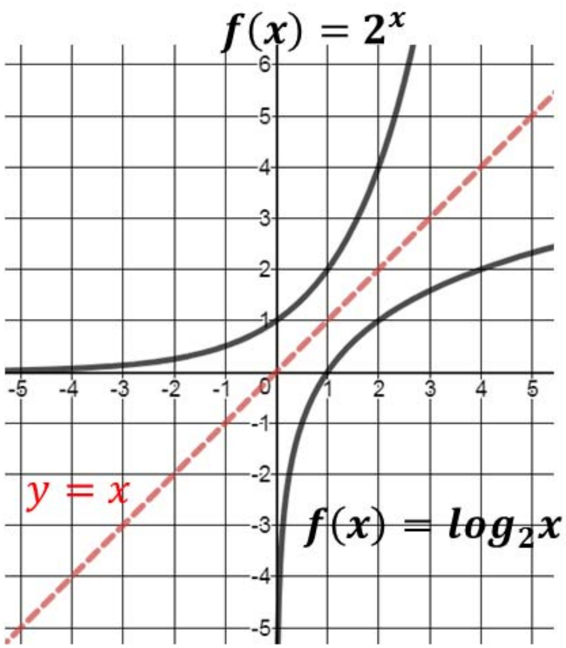
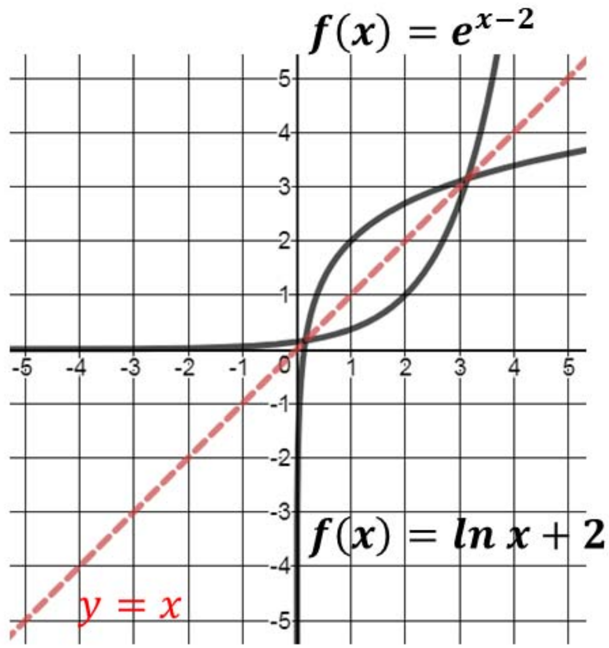
Concept

The graph of a function or relation and its inverse will always be reflections of each other over the line $y = x$. All points of intersection between a function or relation and its inverse will be on the line $y = x$.





Objective: Find the inverse of an exponential function

Concept

The inverse of an exponential function is a logarithmic function.

Steps to Find the Inverse Function of an Exponential Function

1. Change the function notation to y.

2. Interchange (switch) the x and y variables. Do not move any numbers or other symbols.

3. Use algebra to solve for the y variable.
   a. Isolate the power expression.
   b. Write in logarithmic form.
   c. Solve for the y variable, if necessary.

4. Rewrite the y variable using inverse notation.

Objective: Find the inverse of an exponential function

Ex) Find the inverse of each function.

$$f(x) = e^{x+3}$$

1. $y = e^{x+3}$

2. a) $x = e^{y+3}$

3. b) $\log_e x = y + 3$

   c) $\ln x = y + 3$
   $\quad -3 \qquad -3$

   $y = \ln x - 3$

4. $\boxed{f^{-1}(x) = \ln x - 3 \quad \text{or} \quad f^{-1}(x) = \ln(x) - 3}$

Objective: Find the inverse of an exponential function

Ex) Find the inverse of each function.

$$g(x) = 2(10)^{x-5}$$

1. $y = 2(10)^{x-5}$

2. $x = 2(10)^{y-5}$

(a) $\frac{x}{2} = \frac{2(10)^{y-5}}{2}$

3. $\frac{x}{2} = (10)^{y-5}$

(b) $\log_{10} \frac{x}{2} = y - 5$

(c) $\log \frac{x}{2} = y - 5$

$\quad +5 \quad\quad +5$

$y = \log \frac{x}{2} + 5$

4. $\boxed{g^{-1}(x) = \log \frac{x}{2} + 5 \quad \text{or} \quad g^{-1}(x) = \log\left(\frac{x}{2}\right) + 5}$

Objective: Find the inverse of an exponential function

Ex) Find the inverse of each function.

$$k(x) = e^{3x} + 5$$

1 $y = e^{3x} + 5$

2 $x = e^{3y} + 5$

a $-5 \qquad -5$

3 $\underbrace{x - 5}_{1} = e^{3y}$

b $\log_e(x-5) = 3y$

c $\frac{1}{3} \cdot \ln(x-5) = \frac{3y}{3}$

$y = \frac{1}{3}\ln(x-5)$

4 $\boxed{k^{-1}(x) = \frac{1}{3}\ln(x-5)}$

Objective: Find the inverse of an exponential function

Ex) Find the inverse of each function.

$$p(x) = 7(2)^{5x+8} - 3$$

① $y = 7(2)^{5x+8} - 3$

② $x = 7(2)^{5y+8} - 3$

ⓐ $+3 \qquad +3$

③ $\frac{x+3}{7} = \frac{7(2)^{5y+8}}{7}$

$\frac{x+3}{7} = 2^{5y+8}$

ⓑ $\log_2 \frac{x+3}{7} = 5y+8$

ⓒ $-8 \qquad -8$

$\frac{\log_2 \frac{x+3}{7} - 8}{5} = \frac{5y}{5}$

$y = \frac{1}{5}\log_2 \frac{x+3}{7} - \frac{8}{5}$

④ $\boxed{p^{-1}(x) = \frac{1}{5}\log_2 \frac{x+3}{7} - \frac{8}{5} \quad \text{or} \quad p^{-1}(x) = \frac{1}{5}\log_2\left(\frac{x+3}{7}\right) - \frac{8}{5}}$

Objective: Find the inverse of an exponential function

Closure

Find the inverse of the logarithmic function $f(x) = \log(x + 4)$.

$$y = \log_{10}(x+4)$$

$$x = \log_{10}(y+4)$$

$$10^x = y+4$$

$$y = 10^x - 4$$

$$\boxed{f^{-1}(x) = 10^x - 4}$$