Objective: Find Trigonometric Values of Angles Coterminal to Unit Circle Angles

Concept

Steps to Find Trigonometric Values of Angles Coterminal to Unit Circle Angles

1. Find the coterminal angle in the interval $[0, 2\pi)$. This should be a Unit Circle angle.

2. Determine the Quadrant in which the angle in step 1 terminates and the sign of the trigonometric ratio in this quadrant. If the angle is quadrantal, go to step 4.

3. Find the reference angle for the angle in step 1.

4. Find the trigonometric value of the reference angle or quadrantal angle. Use a Reciprocal Identity or Quotient Identity if necessary.

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Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

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Ex) Use the Unit Circle and Trigonometric Identities to find each exact value.

$$\sin\frac{8\pi}{3} =$$

① $\frac{8\pi}{3} = 2\frac{2}{3}\pi - 2\pi = \frac{2}{3}\pi = \frac{2\pi}{3}$

② $\sin\frac{8\pi}{3} = \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

③ $\sin\frac{8\pi}{3} = \frac{\sqrt{3}}{2}$

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Ex) Use the Unit Circle and Trigonometric Identities to find each exact value.

$$\cos\left(-\frac{11\pi}{4}\right) =$$

① $-\frac{11\pi}{4} = -2\frac{3}{4}\pi + 4\pi$

$= -\frac{11\pi}{4} + \frac{16\pi}{4} = \frac{5\pi}{4}$

② $\cos\left(\frac{-11\pi}{4}\right) = \cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

③ $\cos\left(\frac{-11\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

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Ex) Use the Unit Circle and Trigonometric Identities to find each exact value.

$$\sec\left(\frac{7\pi}{2}\right) =$$

① $\frac{7\pi}{2} = 3\frac{1}{2}\pi - 2\pi = 1\frac{1}{2}\pi = \frac{3\pi}{2}$

② $\sec\left(\frac{7\pi}{2}\right) = \sec\frac{3\pi}{2} = \frac{1}{\cos\frac{3\pi}{2}} = \frac{1}{0}$

$=$ undefined

③ $\sec\left(\frac{7\pi}{2}\right) =$ undefined

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Ex) Use the Unit Circle and Trigonometric Identities to find each exact value.

$$\cot\left(-\frac{7\pi}{6}\right) =$$

① $-\frac{7\pi}{6} = -1\frac{1}{6}\pi + 2\pi$

$= -\frac{7\pi}{6} + \frac{12\pi}{6} = \frac{5\pi}{6}$

② $\cot\left(\frac{-7\pi}{6}\right) = \cot\frac{5\pi}{6} = \frac{\cos\frac{5\pi}{6}}{\sin\frac{5\pi}{6}}$

$= \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2}\cdot\frac{2}{1} = -\sqrt{3}$

③ $\cot\left(\frac{-7\pi}{6}\right) = -\sqrt{3}$

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Closure

Amy was finding $\sin\left(\frac{11\pi}{3}\right)$. Her first step was to find the angle coterminal with $\frac{11\pi}{3}$ in the interval $[0,2\pi)$. Her work is shown. Explain why her angle is incorrect and determine the correct coterminal angle.

$$\frac{11\pi}{3} = 3\frac{2}{3}\pi \rightarrow 3\frac{2}{3}\pi - 3\pi = \frac{2}{3}\pi \rightarrow \frac{2\pi}{3}$$

Amy's angle is incorrect because she subtracted 3π which is 1.5 revolutions, so these angles do not have the same terminal side. $\frac{2\pi}{3}$ is in the second quadrant. $\frac{11\pi}{3}$ is in the fourth quadrant. A coterminal angle is always one or more full revolutions. Amy should have only subtracted 2π. The correct coterminal angle is $\frac{5\pi}{3}$.