Objective: Graph Exponential Functions with Base e and find key features.

Prior Knowledge

What is the next term in the sequence?

3, 7, 11, 15, 19, __23__

$n!$ is read "n factorial", where n is any whole number
$n!$ is equal to the product of all whole numbers from n down to 1.
$0!$ is defined as 1.

$0! = 1$
$1! = 1$
$2! = 2 \cdot 1 = 2$
$3! = 3 \cdot 2 \cdot 1 = 6$

Find the following.

$4! = \quad 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$5! = \quad 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

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Concept

In Math I you learned about sequences. A series is when the terms of a sequence are added together.

Consider the Maclaurin Series:

$$\sum_{k=0}^{\infty} \frac{1}{k!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + \ \ldots$$

$$= 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720} + \frac{1}{5040} + \ \ldots$$

Use a calculator to add the numbers in the series together. What value do you get?

2.7182539683 ...

As more terms in the series are added together, **the sum approaches the irrational number e**. The irrational number e is sometimes called the **natural base** and is used extensively in scientific and other applications of exponential growth and decay.

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Identifying Base e Functions as Exponential Growth or Exponential Decay.

Base e exponential growth: $f(x) = a \cdot e^{x-h} + k$

Base e exponential decay: $f(x) = a \cdot e^{-x+h} + k$

What to include in the graph of an exponential function.

- **horizontal asymptote**; Note: for the functions $f(x) = a \cdot e^{-x+h} + k$
- and $f(x) = a \cdot e^{x-h} + k$, the horizontal asymptote is $\boldsymbol{y = k}$.
- **key points**, including the ***y*-intercept** and/or **zero** when reasonable
- **end behavior**

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Identify each function as exponential growth or exponential decay.

$f(x) = e^{x+4} - 2$

Base = e >1
exponential growth

$g(x) = -2e^{x} - 6$

Base = e >1
exponential growth

$h(x) = e^{-x} + 5$

Base = $\frac{1}{e}$ <1
exponential decay

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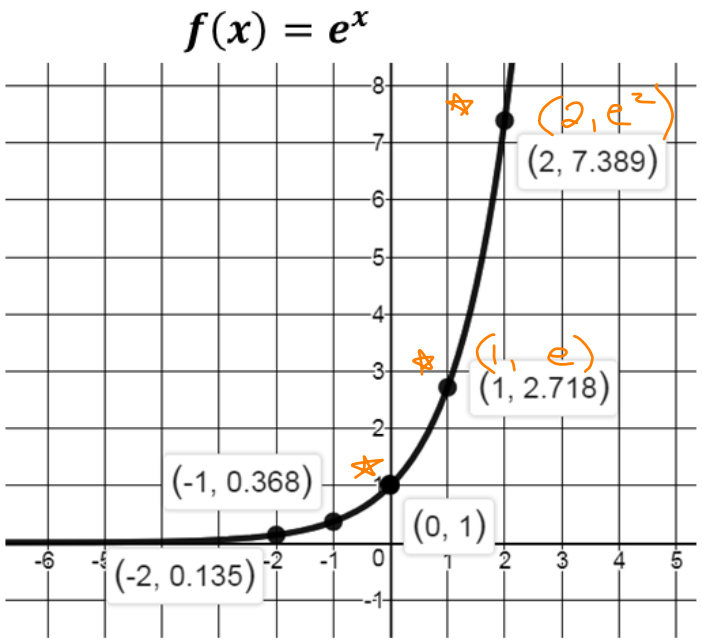
Concept

The function $f(x) = e^x$ is the parent function for base e exponential functions.

x	$f(x) = e^x$
-2	$e^{-2} = \frac{1}{e^2} \approx 0.135$
-1	$e^{-1} \approx 0.368$
0	$e^0 = 1$
1	$e^1 \approx 2.718$
2	$e^2 \approx 7.389$



Domain: the set of all real numbers; $\{x|-\infty < x < +\infty\}$; $(-\infty, +\infty)$
Range: $\{y|y > 0\}$; $(0, +\infty)$
Horizontal Asymptote: $y = 0$
End Behavior: $as\ x \to -\infty, f(x) \to 0; as\ x \to +\infty, f(x) \to +\infty$

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Ex) Graph the function. State the key features.

$g(x) = e^{-x}$ base $= e^{-1} = \frac{1}{e} < 1$

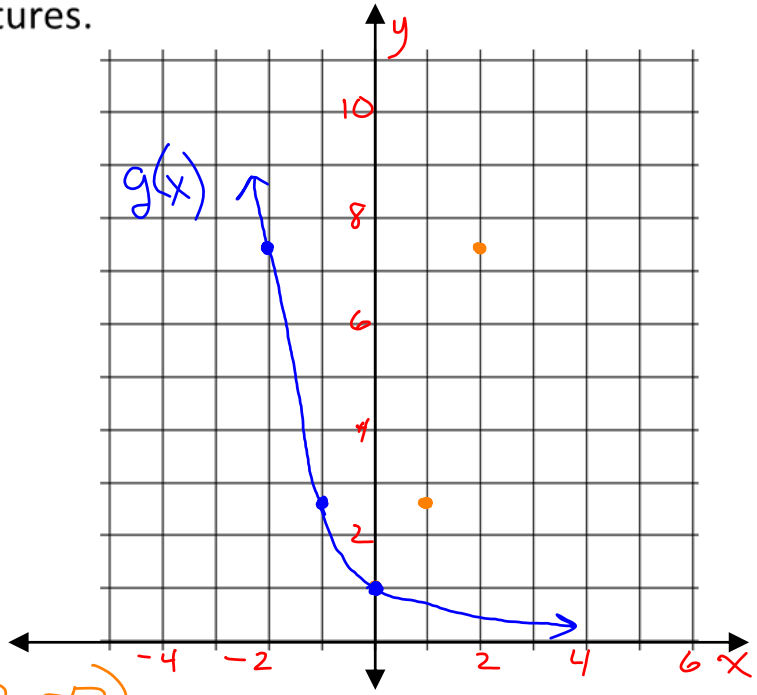
$g(x) = ae^{\frac{1}{b}(x-h)} + k$

$a = 1$ none

$\frac{1}{b} = -1$ y-axis refl.

$h = 0$ none

$k = 0$ none



Exponential Growth/Decay (circle one)

Domain: $\{x \mid -\infty < x < \infty\}$ / $(-\infty, \infty)$
(set notation) (interval notation)

Range: $\{y \mid y > 0\}$ / $(0, \infty)$
(set notation) (interval notation)

Horizontal Asymptote: $y = 0$

End Behavior: as $x \to -\infty$, $g(x) \to +\infty$
as $x \to +\infty$, $g(x) \to 0$

Objective: Graph Exponential Functions with Base e and find key features.

Ex) Graph the function. State the key features.

$g(x) = 2e^{x-2} - 2$

base $= e^1 = e > 1$

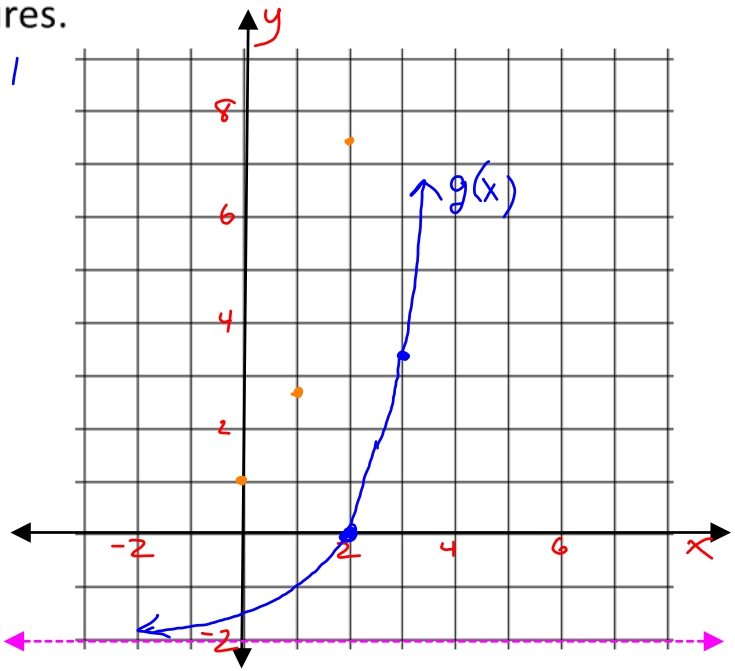
$g(x) = 2e^{\frac{1}{b}(x-h)} + k$

$a = 2$ vertical stretch

$\frac{1}{b} = 1$ none

$h = 2$ right 2

$k = -2$ down 2



Exponential Growth/Decay (circle one)

Domain: $\{x \mid -\infty < x < \infty\}$ / $(-\infty, \infty)$
(set notation) (interval notation)

Range: $\{y \mid y > -2\}$ / $(-2, \infty)$
(set notation) (interval notation)

Horizontal Asymptote: $y = -2$

End Behavior: as $x \to -\infty$, $g(x) \to -2$
as $x \to +\infty$, $g(x) \to +\infty$

Objective: Graph Exponential Functions with Base e and find key features.

Ex) Graph the function. State the key features.

$g(x) = -e^{-x-3} + 1$ base $= e^{-1} = \frac{1}{e} < 1$

$g(x) = -e^{-1(x+3)} + 1$

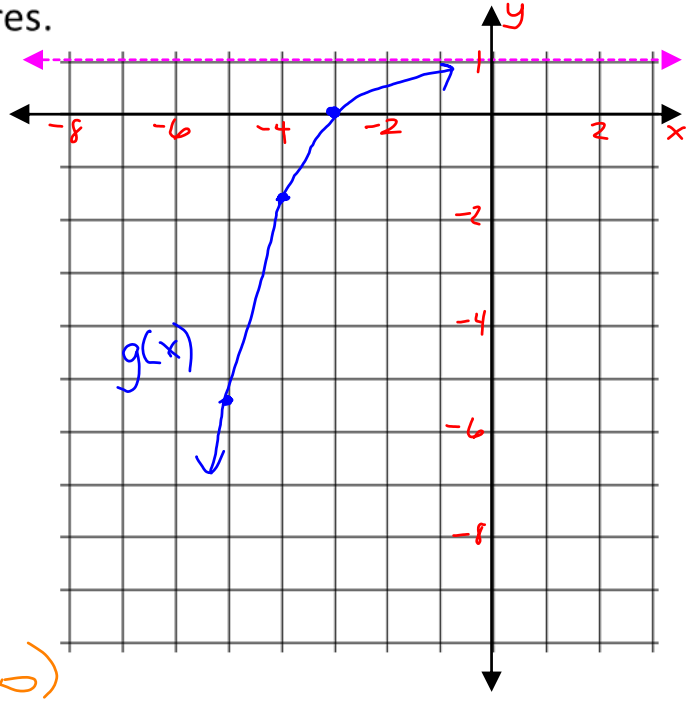
$g(x) = ae^{\frac{1}{b}(x-h)} + k$

$a = -1$ x-axis refl.

$\frac{1}{b} = -1$ y-axis refl.

$h = -3$ left 3

$k = 1$ up 1



Exponential Growth/Decay (circle one)

Domain: $\{x \mid -\infty < x < \infty\}$ (set notation) / $(-\infty, \infty)$ (interval notation)

Range: $\{y \mid y < 1\}$ (set notation) / $(-\infty, 1)$ (interval notation)

Horizontal Asymptote: $y = 1$

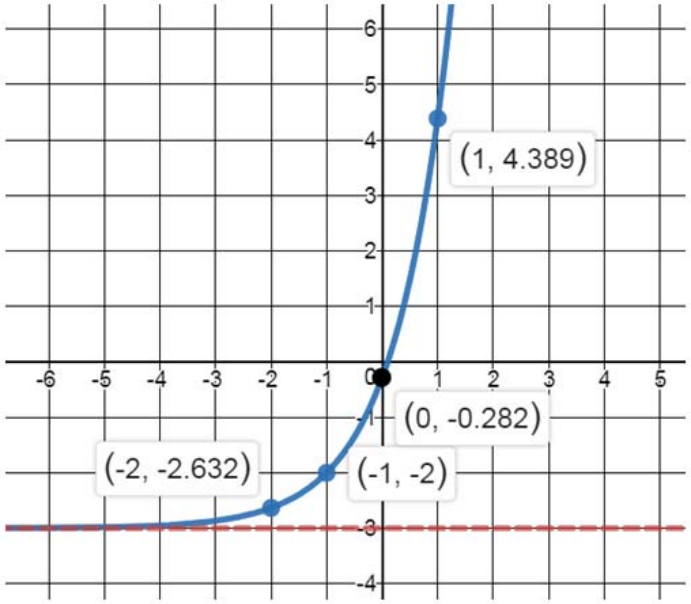
End Behavior: as $x \to -\infty$, $g(x) \to -\infty$

as $x \to +\infty$, $g(x) \to 1$

Objective: Graph Exponential Functions with Base e and find key features.

Practice) Graph the function. State the key features.

$g(x) = e^{x+1} - 3$



Exponential Growth/Decay (circle one)

Domain: $\{x|-\infty < x < +\infty\}$ / $(-\infty, +\infty)$
(set notation) (interval notation)

Range: $\{y|y > -3\}$ / $(-3, +\infty)$
(set notation) (interval notation)

Horizontal Asymptote: $y = -3$

End Behavior: $as\ x \to -\infty, g(x) \to -3$

$as\ x \to +\infty, g(x) \to +\infty$

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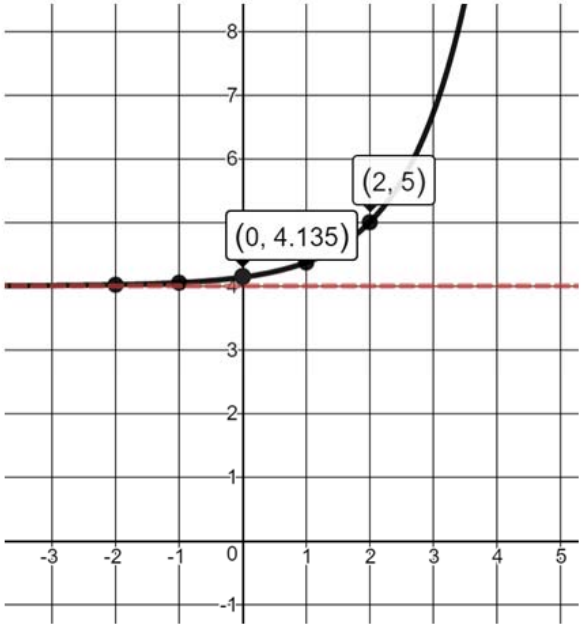
Closure

Using the graph, what is the value of k for the function $h(x) = e^{x-2} + k$?



$k = 4$