Objective: Simplify Numerical Expressions with Rational Exponents

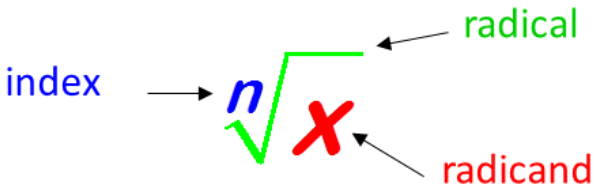
Concept

Rational and irrational numbers expressed in radical form can also be expressed with fractional exponents. When the number has a fractional exponent, it is said to be in rational exponent form.

$$b^{\frac{1}{n}} = \sqrt[n]{b}$$

and

$$b^{\frac{p}{n}} = \sqrt[n]{b^p} \text{ or } b^{\frac{p}{n}} = \left(\sqrt[n]{b}\right)^p$$



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Concept

Properties of Rational Exponents

For all nonzero real numbers a and b and rational numbers m and n

Words	Numbers	Algebra
Product of Powers Property: to multiply powers with the same base, add the exponents	$12^{\frac{1}{2}} \cdot 12^{\frac{3}{2}} = 12^{\frac{1}{2}+\frac{3}{2}} = 12^2 = 144$	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers Property: to divide powers with the same base, subtract the exponents	$\frac{125^{\frac{2}{3}}}{125^{\frac{1}{3}}} = 125^{\frac{2}{3}-\frac{1}{3}} = 125^{\frac{1}{3}} = 5$	$\frac{a^m}{a^n} = a^{m-n}$ $or \frac{a^m}{a^n} = \frac{1}{a^{n-m}}$
Power of a Power Property: to raise one power to another, multiply the exponents	$\left(8^{\frac{2}{3}}\right)^3 = 8^{\frac{2}{3}\cdot 3} = 8^2 = 64$	$(a^m)^n = a^{m \cdot n}$
Power of a Product Property: to find a power of a product, distribute the exponent	$(16 \cdot 25)^{\frac{1}{2}} = 16^{\frac{1}{2}} \cdot 25^{\frac{1}{2}} = 4 \cdot 5 = 20$	$(ab)^m = a^m b^m$
Power of a Quotient Property: to find a power of a quotient, distribute the exponent	$\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}} = \frac{2}{3}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

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Properties of Rational Exponents		
For all nonzero real numbers a and b and rational numbers m and n		
Words	**Numbers**	**Algebra**
Negative Exponent Property: moving a power from numerator to denominator or vice versa changes the sign on the exponent	$36^{-\frac{1}{2}} = \frac{1}{36^{\frac{1}{2}}} = \frac{1}{6}$ $\frac{1}{36^{-\frac{1}{2}}} = \frac{36^{\frac{1}{2}}}{1} = \frac{6}{1} = 6$	$a^{-n} = \frac{1}{a^n}$ or $\frac{1}{a^{-n}} = a^n$
Zero Exponent Property: any monomial to a power of 0 is equal to 1	$(3)^0 = 1$	$(a)^0 = 1$

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Ex) Simplify each numerical value as much as possible.

$(125)^{\frac{4}{3}}$ ← exponent, 4/3 index

$= 125^{4/3}$

$= \left(\sqrt[3]{125}\right)^4$

$= 5^4$

$= \boxed{625}$

$\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

$= \left(\frac{81}{16}\right)^{\frac{-3}{4}}$ exponent, index

$= \frac{\left(\sqrt[4]{81}\right)^{-3}}{\left(\sqrt[4]{16}\right)^{-3}}$

$= \frac{(3)^{-3}}{(2)^{-3}} = \frac{2^3}{3^3}$

$= \boxed{\frac{8}{27}}$

Objective: Simplify Numerical Expressions with Rational Exponents

Ex) Simplify each numerical value as much as possible.

$$25^{\frac{3}{5}} \cdot 25^{\frac{7}{5}}$$

product of powers

$$= 25^{\frac{3}{5}+\frac{7}{5}}$$

$$= 25^{\frac{10}{5}}$$

$$= 25^{2}$$

$$= \boxed{625}$$

Objective: Simplify Numerical Expressions with Rational Exponents

Ex) Simplify each numerical value as much as possible.

$$\frac{8^{\frac{1}{2}}}{8^{\frac{5}{6}}}$$

quotient of powers

$$\frac{a^m}{a^n} = a^{m-n} \quad (a)$$

$$= \frac{1}{a^{n-m}} \quad (b)$$

(a) $8^{\frac{3}{3}\cdot\frac{1}{2} - \frac{5}{6}} = 8^{\frac{3}{6} - \frac{5}{6}}$

$= 8^{-\frac{2}{6}} = 8^{-\frac{1}{3}}$

$= \left(\sqrt[3]{8}\right)^{-1} = 2^{-1} = \boxed{\frac{1}{2}}$

(b) $\frac{1}{8^{\frac{5}{6}-\frac{1}{2}\cdot\frac{3}{3}}} = \frac{1}{8^{\frac{5}{6}-\frac{3}{6}}}$

$= \frac{1}{8^{\frac{2}{6}}} = \frac{1}{8^{\frac{1}{3}}}$

$= \frac{1}{\sqrt[3]{8}} = \boxed{\frac{1}{2}}$

Objective: Simplify Numerical Expressions with Rational Exponents

Ex) Simplify each numerical value as much as possible.

$$\left(32^{\frac{4}{3}}\right)^{-\frac{3}{5}}$$

power of a power

$$32^{\frac{4}{3}\cdot\frac{-3}{5}}$$

$$= 32^{\frac{-4}{5}} = \left(\sqrt[5]{32}\right)^{-4}$$

$$= 2^{-4} = \frac{1}{2^4}$$

$$= \boxed{\frac{1}{16}}$$

Objective: Simplify Numerical Expressions with Rational Exponents

Closure

Darlene simplified an expression with rational exponents. Her work is shown. Identify the step where Darlene made a mistake. What was her mistake and what is the correct answer?

$simplify \quad 8^{\frac{5}{12}} \cdot 8^{\frac{1}{4}}$

$step\,1: \quad 8^{\frac{5}{12}} \cdot 8^{\frac{3}{12}}$

$step\,2: \quad 64^{\frac{8}{12}}$

$step\,3: \quad 64^{\frac{2}{3}}$

$step\,4: \quad \left(\sqrt[3]{64}\right)^2$

$step\,5: \quad (4)^2$

$step\,6: \quad \boxed{16}$

Darlene made a mistake in step 2. She shouldn't have multiplied the 8s. The correct answer is 4.

$simplify \quad 8^{\frac{5}{12}} \cdot 8^{\frac{1}{4}}$

$step\,1: \quad 8^{\frac{5}{12}} \cdot 8^{\frac{3}{12}}$

$step\,2: \quad 8^{\frac{8}{12}}$

$step\,3: \quad 8^{\frac{2}{3}}$

$step\,4: \quad \left(\sqrt[3]{8}\right)^2$

$step\,5: \quad (2)^2$

$step\,6: \quad \boxed{4}$