Objective: Solve context problems using logarithms.

Concept

Steps to Solve an Exponential Equation Using Logarithms

1. Write the equation with a single power on each side.
2. Take the logarithm of both sides. (Common Logarithm or Natural Logarithm)
3. Use the Power Property of Logarithms: $\log_b a^x = x log_b a$
4. Solve this equation using algebra. Approximate if necessary.
5. State the solution to the exponential equation. Interpret in terms of the context.

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Ex) Emily deposits \$250 into an account that pays 4.5% compounded quarterly. The formula $A = P(1.01125)^{4t}$ gives the amount A in the account after t years for an initial investment P.

a) What is the balance of the account after 8 months?

$A = ?$ when $t = 8\text{mo}$

$= \frac{8\cancel{\text{mo}}}{1} \cdot \frac{1\text{yr}}{12\cancel{\text{mo}}}$

$t = \frac{8}{12}$ yr

$A = 250(1.01125)^{(4 \cdot 8/12)}$

$= \$257.57$

The balance of the account after 8 months is \$257.57.

b) How long it will take, in years and months, for the balance in the account to double? Round to the nearest month.

$t = ?$ when A doubles

$A = 2(250) = 500$

$\frac{500}{250} = \frac{250(1.01125)^{4t}}{250}$

$2 = 1.01125^{4t}$

$\ln 2 = \ln 1.01125^{4t}$

$\frac{\ln 2}{(4 \cdot \ln 1.01125)} = \frac{4t \cdot \ln 1.01125}{(4 \cdot \ln 1.01125)}$

$t = \frac{\ln(2)}{(4 \cdot \ln(1.01125))} = 15.489\ldots$ yrs

$= 15\text{yr} \; \frac{.489\ldots\text{yr}}{1} \cdot \frac{12\text{mo}}{1\text{yr}}$

≈ 15yr 6mo

The balance in the account will double in about 15 years 6 months.

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Ex) Ajay made a steaming pot of stew. When the temperature of the stew reached $70°\ C$, Ajay turned the stove off and the stew began cooling. The temperature, T (measured in degrees Celsius), of the stew, m minutes since Ajay turned the stove off can be modeled by the function $T(m) = 20 + 50(10)^{-0.04m}$.

a) How long, to the nearest minute, did it take the stew to reach a temperature of $43°$ C?

$m = ?$ when $T = 43°C$

$T = 20 + 50(10)^{-.04m}$

$43 = 20 + 50(10)^{-.04m}$

$-20 \quad -20$

$\frac{23}{50} = \frac{50(10)^{-.04m}}{50}$

$0.46 = 10^{-.04m}$

$\log 0.46 = \log 10^{-.04m}$

$\log 0.46 = -0.04m \log 10$

$-0.04m \cdot 1$ $\quad \log_{10} 10 = 1$

$\frac{\log 0.46}{-0.04} = \frac{-0.04m}{-0.04}$

$m \approx 8.43\ldots$ minutes

≈ 8 min

The stew reaches 43°C in about 8 minutes.

b) What is the temperature of the stew, to the nearest degree, after 5 minutes?

$T = ?$ when $m = 5$ min.

$T = 20 + 50(10)^{-.04(5)}$

$= 51.547\ldots\ °C$

$\approx 52°C$

After 5 minutes, the temperature of the stew is about 52°C.

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Ex) A biologist in Nepal is studying the local population of red pandas, a vulnerable species that may soon be endangered. The biologist has determined that the number of red pandas, P, in the local population t years since the species has been monitored can be modeled by the function $P(t) = 800e^{0.0416t}$. When, to the nearest tenth of a year, will the red panda population in this area reach 2000?

$t = ?$ when $P = 2000$

$P = 800e^{0.0416t}$

$\frac{2000}{800} = \frac{800e^{0.0416t}}{800}$

$2.5 = e^{0.0416t}$

$\ln 2.5 = \ln e^{0.0416t}$

$\ln 2.5 = 0.0416t \cdot \underbrace{\ln e}_{1}$

$\frac{\ln 2.5}{0.0416} = \frac{0.0416t}{0.0416}$

$t = \frac{\ln 2.5}{0.0416}$

≈ 22.0 yr

The red panda population in this area will reach 2000 in about 22.0 years.

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Ex) The distance an object travels in feet is modeled by the function $d(m) = 17\ln(3m+6)$, where m is in minutes. What is the average speed of the object during the first 5 minutes the object is in motion? Round to three decimal places.

A.R.O.C. from 0 to 5 min

$$\frac{\Delta d}{\Delta m} = \frac{(17\ln(21)\text{ ft} - 17\ln(6)\text{ ft})}{(5\text{ min} - 0\text{ min})} \approx 4.259 \text{ ft/min}$$

feet per minute

$\boxed{\text{The object's average speed during the first 5 minutes of motion is about 4.259 feet per minute.}}$

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Closure

Determine which type of logarithm is best to use when solving each of the following equations.

1) $70 = 45e^{0.056t}$

b) Natural Logarithm

2) $70 = 10 + 45(3.2)^{0.056t}$

c) Neither is best, either can be used

3) $70 = 45(10)^{0.056t}$

a) Common Logarithm

a) Common Logarithm

b) Natural Logarithm

c) Neither is best; either can be used

d) Logarithms aren't needed to solve the equation