Objective: Solve rational equations algebraically.

Concept

Rational Equation: an equation that involves at least one rational expression (the variable is in at least one denominator).

Example

$$\frac{3x}{x+1} = \frac{2}{x-2} + \frac{x-4}{x^2-x-2}$$

Rational equations can have **extraneous solutions**: solutions that **are excluded values of the expressions in the equation**. For a rational equation, **extraneous solutions are values that create a 0 in one or more denominator**. Extraneous solutions are not included in the final solution set.

For $\frac{3x}{x+1} = \frac{2}{x-2} + \frac{x-4}{x^2-x-2}$, a solution of -1 or 2 would be extraneous and would not be included in the final solution set.

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Steps to Solve a Rational Equation

1. Find the LCD (lowest common denominator).
2. Multiply every term by the LCD to clear the denominators (use the multiplication property of equality).
3. Solve the resulting equation.
4. Check for Extraneous Solutions
   (excluded values; solutions that make the denominator equal to 0).
5. State the final solution set.

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Ex) Solve the equation.

$$\frac{-3x+4}{x^2-7x+12} = \frac{x+2}{x-3} + \frac{2}{x-4}$$

$(x-4)(x-3)$

① LCD

$= (x-3)(x-4)$

②

$$\frac{(-3x+4)}{\cancel{(x-4)}\cancel{(x-3)}} \cdot \frac{\cancel{(x-3)}\cancel{(x-4)}}{1} = \frac{(x+2)}{\cancel{(x-3)}} \cdot \frac{\cancel{(x-3)}(x-4)}{1} + \frac{2}{\cancel{(x-4)}} \cdot \frac{(x-3)\cancel{(x-4)}}{1}$$

③ solve

$-3x+4 = x^2 - \cancel{2x} - 8 + \cancel{2x} - 6$

★ quadratic equation

$-3x + 4 = x^2 - 14$

$+3x \quad -4 \qquad +3x \quad -4$

★ standard form

$0 = x^2 + 3x - 18$

$0 = (x+6)(x-3)$

★ zero product property

$x+6=0$ or $x-3=0$

$-6 \quad -6 \qquad +3 \quad +3$

$x = -6 \qquad \cancel{x = 3}$ extraneous

④ check for extraneous solutions

⑤ $\boxed{x = -6}$

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Ex) Solve the equation.

$$\frac{2x-9}{(x-7)} + \frac{x}{2} = \frac{5}{x-7}$$

① LCD $= 2(x-7)$

② $\frac{(2x-9)}{\cancel{(x-7)}} \cdot \frac{2\cancel{(x-7)}}{1} + \frac{x}{\cancel{2}} \cdot \frac{\cancel{2}(x-7)}{1} = \frac{5}{\cancel{(x-7)}} \cdot \frac{2\cancel{(x-7)}}{1}$

③ solve.

$$4x-18+x^2-7x=10$$

$$x^2-3x-18=10$$

$$-10 \quad -10$$

$$x^2-3x-28=0$$

$$(x-7)(x+4)=0$$

$x-7=0$ or $x+4=0$

$\cancel{x=7}$ extraneous $\quad x=-4$

④ check

⑤ $\boxed{x=-4}$

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Ex) Solve the equation.

$$\frac{56}{x^2-2x-15}-\frac{6}{x+3}=\frac{7}{x-5}$$

$(x-5)(x+3)$

① LCD

$=(x-5)(x+3)$

② first

$$\frac{56}{\cancel{(x-5)}\cancel{(x+3)}}\cdot\frac{\cancel{(x-5)}\cancel{(x+3)}}{1}-\left[\frac{6}{\cancel{(x+3)}}\cdot\frac{(x-5)\cancel{(x+3)}}{1}\right]=\frac{7}{\cancel{(x-5)}}\cdot\frac{\cancel{(x-5)}(x+3)}{1}$$

③ solve

$$56-(6x-30)=7x+21$$

*linear

$$56-6x+30=7x+21$$
$$+6x\quad -21\qquad +6x\quad -21$$
$$\frac{65}{13}=\frac{13x}{13}$$
$$\cancel{x=5}$$

④ check for extraneous solutions

⑤ $\boxed{\text{no solution } \varnothing}$

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Closure

Jasmine solved a rational equation. Her work is shown. Did Jasmine state the solution correctly? Explain your reasoning.

$$\frac{x}{x+2}+\frac{3}{x}=\frac{-2}{x+2} \qquad LCD = x(x+2)$$

$$\frac{x}{(x+2)}\cdot\frac{x(x+2)}{1}+\frac{3}{x}\cdot\frac{x(x+2)}{1}=\frac{-2}{(x+2)}\cdot\frac{x(x+2)}{1}$$

$$x\cdot x+3(x+2)=-2x$$

$$x^2+3x+6=-2x$$

$$x^2+5x+6=0$$

$$(x+2)(x+3)=0$$

$$x+2=0 \quad x+3=0$$

$$x=-2 \quad x=-3$$

$$\boxed{solution: x=-3,-2}$$

Jasmine did not state the solution correctly. The solution of -2 is extraneous because it creates a 0 in two denominators. The correct solution is $x = -3$.