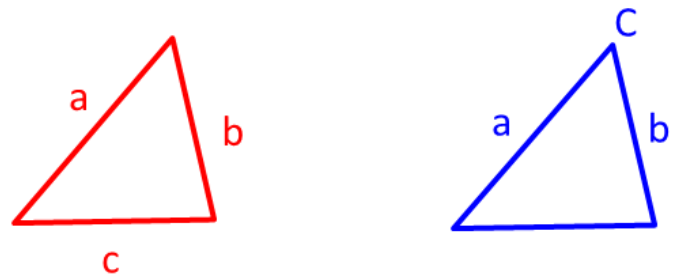
Objective: Use the Law of Cosines to Solve Non-Right Triangles

Concept

For triangles where SSS or SAS is known, the Law of Sines cannot be used.

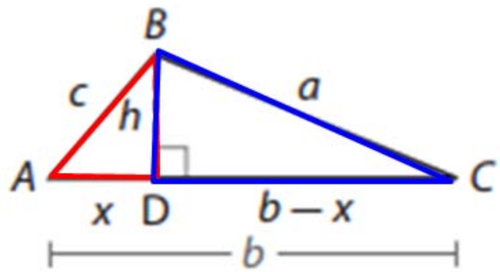


In these two situations, the Law of Cosines must be used.

Objective: Use the Law of Cosines to Solve Non-Right Triangles

Concept

To derive the Law of Cosines, draw $\triangle ABC$ with altitude $\overline{BD}$. If x represents the length of $\overline{AD}$, the length of $\overline{DC}$ is $b - x$.



1. Use the Pythagorean Theorem to write a relationship for the side lengths of $\triangle BCD$ and for the side lengths of $\triangle ABD$.

$\underline{\triangle BCD}$

$a^2 = (b - x)^2 + h^2$

$a^2 = b^2 - 2bx + x^2 + h^2$

$\underline{\triangle ABD}$

$c^2 = x^2 + h^2$

2. Substitute $x^2 + h^2$ with c^2.

$$a^2 = b^2 - 2bx + c^2$$

3. In ΔABD, $cosA = \frac{x}{c}$. Solving for x yields:

$$x = c\ cosA$$

4. Substituting for x into the equation in step 2 yields the **Law of Cosines**:

$$a^2 = b^2 - 2b(c\ cosA) + c^2$$

$$\boxed{a^2 = b^2 + c^2 - 2bc\ cosA}$$

Objective: Use the Law of Cosines to Solve Non-Right Triangles

Concept

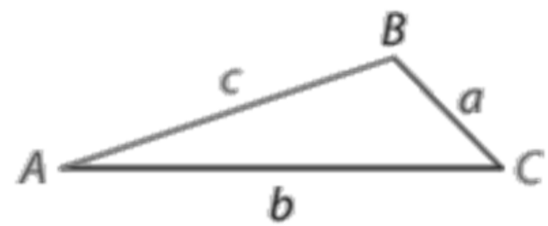
To solve a triangle when SSS or SAS is known, use the Law of Cosines.

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\ cosA$$

$$b^2 = a^2 + c^2 - 2ac\ cosB$$

$$c^2 = a^2 + b^2 - 2ab\ cosC$$



Objective: Use the Law of Cosines to Solve Non-Right Triangles

Procedure for Using the Law of Cosines

When SSS is known:

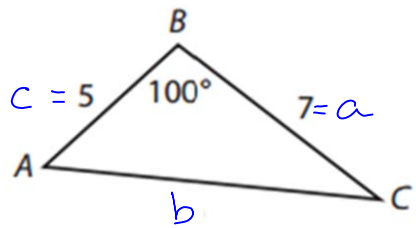
1. Find the largest angle first. This determines whether the triangle is acute or obtuse.
2. Use the Law of Cosines to find the second largest angle.
3. Use the Triangle Sum Theorem to find the third angle.

When SAS is known:

1. Find the side opposite the known angle.
2. Find the largest unknown angle measure using the Law of Cosines.
3. Use the Triangle Sum Theorem to find the third angle.

Objective: Use the Law of Cosines to Solve Non-Right Triangles

Ex) Solve the triangle. Round side measures to the nearest tenth and angle measures to the nearest degree.



① find side b.

$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos B$

$b^2 = 7^2 + 5^2 - 2(7)(5) \cdot \cos 100°$

$b = \sqrt{7^2 + 5^2 - 2(7)(5) \cdot \cos 100°}$

$\boxed{b \approx 9.3 \text{ units}}$

② find m∠A

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos A$

$7^2 = 9.3^2 + 5^2 - 2(9.3)(5) \cdot \cos A$

$\frac{7^2 - 9.3^2 - 5^2}{-2(9.3)(5)} = \frac{\cancel{-2(9.3)(5)} \cdot \cos A}{\cancel{-2(9.3)(5)}}$

$\cos A = \frac{7^2 - 9.3^2 - 5^2}{-2(9.3)(5)}$

$\cancel{\cos^{-1}}(\cancel{\cos} A) = \cos^{-1}\left(\frac{7^2 - 9.3^2 - 5^2}{-2(9.3)(5)}\right)$

$A = \cos^{-1}\left(\frac{(7^2 - 9.3^2 - 5^2)}{(-2(9.3)(5))}\right)$

$\boxed{m\angle A \approx 48°}$

③ find m∠C

$m\angle A + m\angle B + m\angle C = 180°$

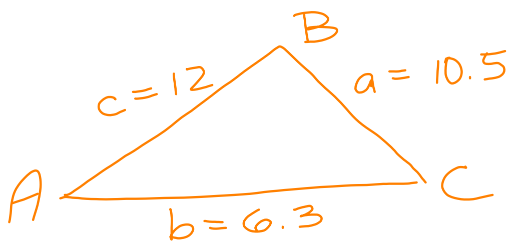
$48° + 100° + m\angle C \approx 180°$

$\boxed{m\angle C \approx 32°}$

Objective: Use the Law of Cosines to Solve Non-Right Triangles

Ex) Solve the triangle. Round side measures to the nearest tenth and angle measures to the nearest degree.

$a = 10.5, b = 6.3, c = 12$



① find m∠C (the largest angle)

$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos C$$

$$12^2 = 10.5^2 + 6.3^2 - 2(10.5)(6.3)\cos C$$

$$12^2 - 10.5^2 - 6.3^2 = -2(10.5)(6.3)\cos C$$

$$\cos C = \frac{12^2 - 10.5^2 - 6.3^2}{-2(10.5)(6.3)}$$

$$C = \cos^{-1}\left(\frac{(12^2 - 10.5^2 - 6.3^2)}{(-2(10.5)(6.3))}\right)$$

$$\boxed{m\angle C \approx 87^\circ}$$

② find m∠A

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos A$$

$$10.5^2 = 6.3^2 + 12^2 - 2(6.3)(12)\cos A$$

$$\cos A = \frac{10.5^2 - 6.3^2 - 12^2}{-2(6.3)(12)}$$

$$A = \cos^{-1}\left(\frac{(10.5^2 - 6.3^2 - 12^2)}{(-2(6.3)(12))}\right)$$

$$\boxed{m\angle A \approx 61^\circ}$$

③ find m∠B

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$61^\circ + m\angle B + 87^\circ \approx 180^\circ$$

$$\boxed{m\angle B \approx 32^\circ}$$

Objective: Use the Law of Cosines to Solve Non-Right Triangles

Closure

What kind of angle measures can the Law of Cosines find? What kind of angle measures can the Law of Sines find?

The Law of Cosines can find both acute and obtuse angle measures.
The Law of Sines can only find acute angle measures.