Objective: Find the inverse of a function in context.

Concept

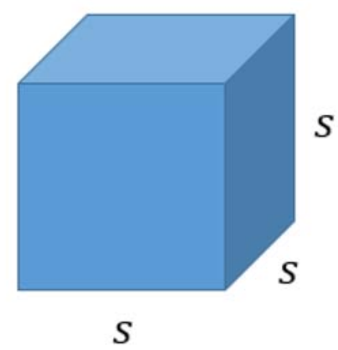
In a context situation, the inverse of a function tells you how to find the value of what was the independent variable in the original function model, given the value of what was the dependent variable.

Function that models the volume of a cube relative to the measure of an edge.

$$V(s) = s^3$$

Inverse Function that models the measure of a cube's edge relative to its volume.

$$s(V) = \sqrt[3]{V}$$



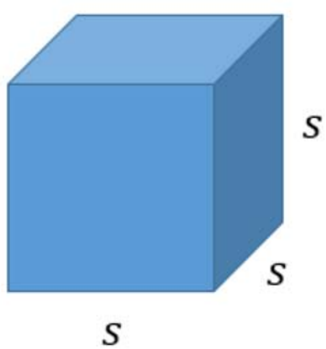
Objective: Find the inverse of a function in context.

Concept

Determining Domain and Range of the Inverse Function in a Real-World Application

As with all functions and their inverse functions, **the domain of an inverse function is the range of the original function** and **the range of the inverse function is the domain of the original function**.

Domains may need to be restricted based on the type of inverse function and so they are reasonable values of the independent variable in the given context.



Original Function: $V(s) = s^3$

Domain: edge measurements greater than 0

Range: volume measurements greater than 0

Inverse Function: $s(V) = \sqrt[3]{V}$

Domain: volume measurements greater than 0

Range: edge measurements greater than 0

Objective: Find the inverse of a function in context.

Concept

The procedure for solving a function or equation for another variable is the same as solving an equation in one variable for the value of the variable. The algebra must be done in the reverse order of the order of operations.

Procedure for Solving a Function or Equation for Another Variable

Undo Operations Outside of Parentheses

1. Undo Addition and Subtraction outside parentheses
2. Undo Multiplication and Division outside parentheses
3. Undo Exponents outside parentheses

Undo Operations Inside Parentheses

1. Undo Addition and Subtraction inside parentheses
2. Undo Multiplication and Division inside parentheses
3. Undo Exponents inside parentheses

Objective: Find the inverse of a function in context.

Ex) The function $d(t) = 16t^2$ gives the distance d in feet that a dropped object falls in t seconds.

range / domain

a) Write the inverse function to find the time in seconds it takes for an object to fall a distance in feet.

Solve $d(t) = 16t^2$ for t

$\frac{d}{16} = \frac{16t^2}{16}$

$t^2 = \frac{d}{16}$

$\sqrt{t^2} = \sqrt{\frac{d}{16}}$

$t = \sqrt{\frac{d}{16}} \rightarrow t(d) = \sqrt{\frac{d}{16}}$

b) Describe the domain and range of the inverse function model. Include restrictions.

domain: the distance in feet that a dropped object falls where d is non-negative (or $d \geq 0$).

range: time in seconds where t is non-negative (or $t \geq 0$).

c) Estimate how long it will take a penny dropped into a well to fall 48 feet. Round to three decimal places. $t = ?$ when $d = 48$ ft

Use $t = \sqrt{\frac{d}{16}} \rightarrow t = \sqrt{\frac{48}{16}} \approx 1.732$ sec.

It will take about 1.732 seconds for a penny dropped into a well to fall 48 feet.

Objective: Find the inverse of a function in context.

Ex) The function $m(L) = 0.00001L^3$ gives the mass m in kilograms of a red snapper of length L centimeters.

(range) (domain)

a) Write the inverse function to find the length of a red snapper in centimeters given its mass in kilograms.

Solve $m(L) = 0.00001L^3$ for L

$$\frac{m}{0.00001} = \frac{0.00001L^3}{0.00001}$$

$$100{,}000m = L^3$$

$$\sqrt[3]{100{,}000m} = \sqrt[3]{L^3}$$

$$L = \sqrt[3]{100{,}000m} \rightarrow L(m) = \sqrt[3]{100{,}000m}$$

b) Describe the domain and range of the inverse function model. Include restrictions.

domain: the mass in kilograms of a red snapper where m is positive $(m>0)$

range: the length in centimeters of a red snapper where L is positive $(L>0)$.

c) Estimate the length of a red snapper given a mass of 3.2 kilograms. Round to three decimal places.

$L = ?$ when $m = 3.2$ kg

$$L = \sqrt[3]{100{,}000(3.2)}$$

$$L \approx 68.399 \text{ cm}$$

The length of a red snapper with a mass of 3.2 kg is about 68.399 cm.