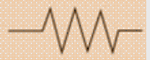
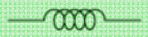
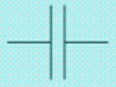
Objective: Apply complex number operations to electrical circuit problems

Concept

Electrical engineers use complex numbers when analyzing electric circuits. An electric circuit commonly contains three types of components: resistors, inductors, and capacitors. As electrons flow through the circuit, each component affects the flow in a different way. The impedance (measured in ohms, Ω) is the resistance to the flow of electrons in an AC circuit.

The table shows the symbol used for each component in a circuit diagram, the phase angle, and the method for representing the impedance of that component as a complex number.

Circuit component	Symbol in circuit diagram	Phase angle	Representation by a complex number
Resistor		0°	A real number *a*
Inductor		90°	An imaginary number *bi* where $b > 0$
Capacitor		-90°	An imaginary number *bi* where $b < 0$

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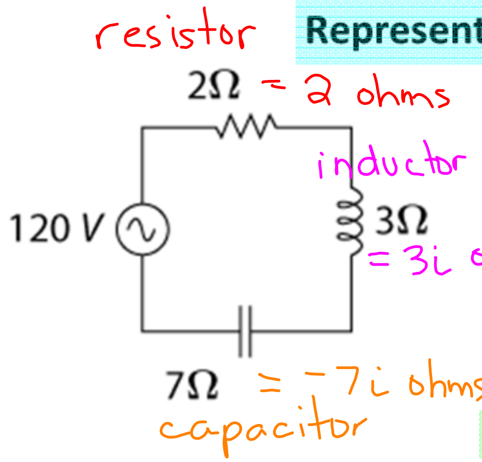
Ex) The diagram of an alternating current (AC) electric circuit is shown, with the impedance of each component. An AC power source, which is shown on the left and labeled 120 V (for volts), causes electrons to flow through the circuit.

Represent the impedance of each component as a complex number.



a) Impedance of the capacitor = $-7i$ ohms.

b) Impedance of the resistor = 2 ohms.

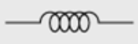
c) Impedance of the inductor = $3i$ ohms.

d) The total impedance of a circuit is the sum of the impedances of its components. The total impedance of the AC circuit is $2-4i$ ohms.

total impedance

$-7i + 2 + 3i$

$2 - 4i$ ohms

Circuit component	Symbol in circuit diagram	Phase angle	Representation by a complex number
Resistor		0°	A real number a
Inductor		90°	An imaginary number bi where $b > 0$
Capacitor		-90°	An imaginary number bi where $b < 0$

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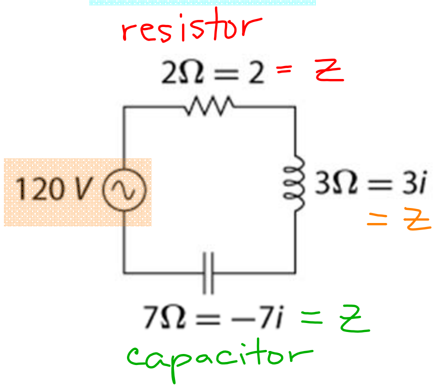
Ex) Ohm's Law for electric circuits says that the voltage V (measured in volts) is the product of the current I (measured in amps) and the impedance Z (measured in ohms).

In the given circuit, the current I is $12 + 24i$ amps.

Calculate the voltage for each component of the circuit.

Ohm's Law

$V = I \cdot Z$



a) What is the voltage for the resistor?

$V = I \cdot Z = (12+24i)2$

$= 2(12+24i) = 24 + 48i$ volts

Voltage for the resistor is $24+48i$ volts

b) What is the voltage for the capacitor?

$V = I \cdot Z = (12+24i)(-7i)$

$= -7i(12+24i) = -84i - 168i^2$

$-168 \cdot -1$

$= 168 - 84i$ volts

The voltage for the capacitor is $168-84i$ volts.

c) What is the voltage for the inductor?

$V = I \cdot Z = (12+24i)3i = 3i(12+24i)$

$= 36i + 72i^2 = -72 + 36i$ volts

$72 \cdot -1$

The voltage of the inductor is $-72+36i$ volts

d) Find the sum of the voltages for the three components of the circuit. What do you notice?

$24+48i + 168-84i + -72+36i$

$24+168+-72 + 48i+-84i+36i$

48
$+36$

$192+-72$

$84i + -84i = 0$

120 volts

The sum of the voltages for the three components equals the voltage of the circuit.

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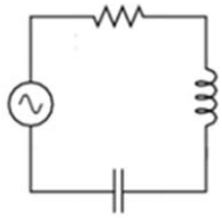
Ex) The current of the AC electric circuit in the diagram is measured as $6+\underset{I}{\underline{4i}}$ amps.

A) What is the total impedance of the circuit?

B) What is the total voltage of the circuit?

ⓐ



9Ω = 9 ohms

4Ω = $4i$ ohms

10Ω = $-10i$ ohms

ⓐ total impedance

$= 9 + 4i + -10i$ ohms

$= 9 - 6i$ ohms

The total impedance of the circuit is $9 - 6i$ ohms.

ⓑ Ohm's Law

Voltage = Current · Impedance

$V = I \cdot Z$

current total impedance

$V = (6+4i)(9-6i)$

$6(9-6i) + 4i(9-6i)$

$54 \underbrace{- \cancel{36i} + \cancel{36i}}_{0} - 24i^2$

$-24 \cdot -1$

$+24$

$= 78$ volts.

The total voltage of the circuit is 78 volts.

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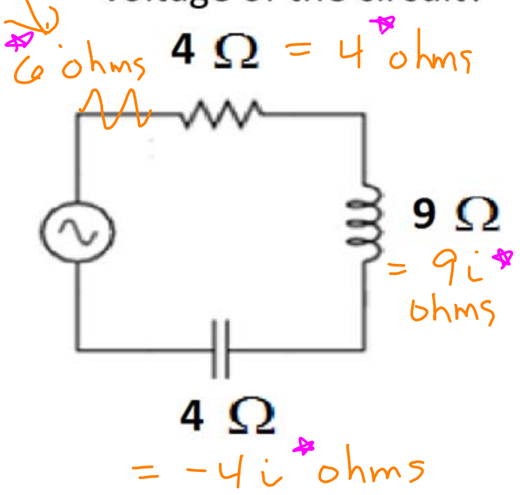
Ex) A second resistor is added to the circuit shown in the diagram. The second resistor has an impedance of 6 ohms.

A) What is the total impedance of the circuit?

B) If the current of the circuit is measured as $16 - 8i$ amps, what is the total voltage of the circuit?



ⓐ $6 + 4 + 9i + -4i = 10 + 5i$ ohms

The total impedance is 10+ 5i ohms.

ⓑ $V = I \cdot Z$

$V = (16 - 8i)(10 + 5i)$

$= 160 + \underset{0}{\cancel{80i} - \cancel{80}i} - \underset{+40}{40i^2}$

= 200 volts

The total voltage is 200 volts.