Objective: Solve problems in context using a quadratic function.
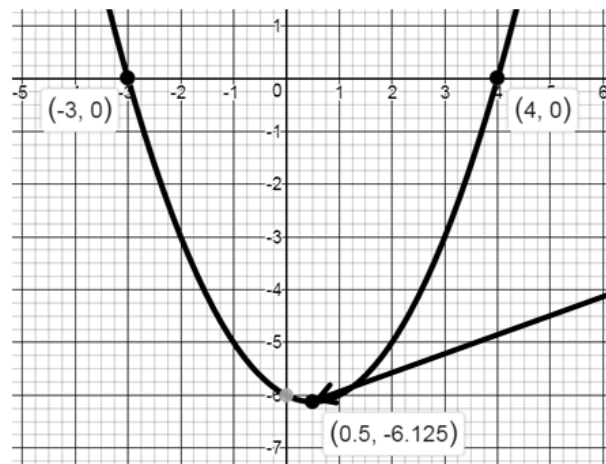
Concept

Zero Product Property

If the product of two or more factors is zero, then at least one of the factors must be equal to zero.

$$\text{If } \quad a \cdot b = 0$$
$$\text{then } a = 0 \quad or \quad b = 0$$

Because of the symmetry of a quadratic function, the x coordinate of the vertex is always halfway between the zeros.

(-3, 0)

(4, 0)

(0.5, -6.125)

$$x = \frac{sum\ of\ zeros}{2} = \frac{-3 + 4}{2} = 0.5$$

Objective: Solve problems in context using a quadratic function.

Ex) A squirrel in a tree drops a chestnut. The function $h(t) = -16(t+2)(t-2)$ models the height in feet of the chestnut as it falls, where t represent time in seconds.
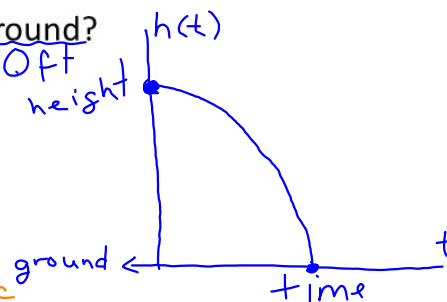
1. When does the chestnut hit the ground?

time=?   height = 0 ft

$h(t) = -16(t+2)(t-2)$

$0 = -16(t+2)(t-2)$

$-16 \neq 0$ $t+2=0$ or $t-2=0$

 $\underset{-2 \; -2}{}$ $\underset{+2 \; +2}{}$

$t = -2 \text{ sec}$ $t = 2 \text{ sec}$

The chestnut hit the ground after 2 seconds.

2. How high is the squirrel when it drops the chestnut?

height =? when time = 0 sec.

$h(0) = -16(0+2)(0-2)$

$= -16(2)(-2)$

$= 64 \text{ ft}$

The squirrel is 64 feet high when it drops the chestnut.

3. What is the height of the chestnut after 1 second?

height =?   time = 1 sec

$h(1) = -16(1+2)(1-2)$

$= -16(3)(-1)$

$= 48 \text{ feet}$

The height of the chestnut after 1 second was 48 feet.

Objective: Solve problems in context using a quadratic function.

Ex) Jonathon is playing fetch with his dog at the park. He throws a ball into the air for his dog to fetch. The function $h(t) = -16(t+0.12)(t-3)$ models the height of the ball in feet, where t represents time in seconds after the ball is released.

1. When Jonathon released the ball, what was its height?
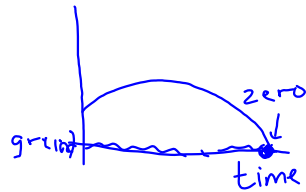
time = 0 sec.   height = ?

$h(0) = -16(0+0.12)(0-3)$
$= -16(0.12)(-3) = 5.76$ ft

When Jonathon released the ball its height was 5.76 feet.

2. How long is the ball in the air?

time = ?   height = on the ground = 0 ft

zero

ground

time

$h(t) = -16(t+0.12)(t-3)$

$0 = -16(t+0.12)(t-3)$

$-16 \neq 0$ $t+0.12 = 0$
               $-0.12 \quad -0.12$

$t-3 = 0$ $t = -0.12$
$+3 \quad +3$
$t = 3$ sec

The ball was in the air for 3 seconds.

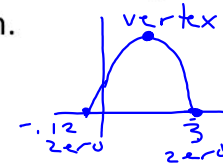3. What is the maximum height the ball reaches? When does it reach this height? Round to the nearest tenth.

$t = h(t)$ vertex

vertex (h, k)

$h = \dfrac{\text{sum of zeros}}{2}$

$-.12$ 3
zero   zero

$= \dfrac{(-.12+3)}{2} = 1.44$ sec
≈ 1.4 sec

$k = h(1.4)$
$= -16(1.4+0.12)(1.4-3)$
$= 38.912$ ft
≈ 38.9 ft

The ball reached a maximum height of about 38.9 feet after about 1.4 sec.

Objective: Solve problems in context using a quadratic function.

Practice) The height of a flare fired from the deck of a ship can be modeled by the function $h(t) = -16(t-6)(t+1)$ where h is the height of the flare above the water in feet and t is the time in seconds.

1. When does the flare hit the surface of the water?

$h(t) = 0 \rightarrow 0 = -16(t-6)(t+1)$

$$t - 6 = 0 \quad \text{or} \quad t + 1 = 0$$

$$t = 6 \qquad \cancel{t = -1}$$

The flare hits the surface of the water after 6 seconds.

2. What is the flare's greatest height? When does the flare reach this height?

vertex = ?

$$t = \frac{-1+6}{2} = 2.5$$

$$h(2.5) = -16(2.5-6)(2.5+1) = 196$$

The flare's greatest height is 196 feet at 2.5 seconds.

3. What is the flare's height after 4 seconds? Is the flare rising or falling at this point in time? Explain your reasoning.

$h(4) = ?$

$h(4) = -16(4-6)(4+1) = 160$

The flare's height after 4 seconds is 160 ft. The flare is falling at this time because it already reached its maximum height at 2.5 seconds.

Objective: Solve problems in context using a quadratic function.

Practice) A dolphin jumps out of the water. The quadratic function

$h(t) = -16t(t-1)$ models the dolphin's height above the water in feet

after t seconds.

1. How long is the dolphin out of the water?

$$t = ?\,\sec, \quad h(t) = 0\,ft$$

$$0 = -16t(t-1)$$

$$-16t = 0 \quad \text{or} \quad t - 1 = 0$$

$$t = 0 \qquad t = 1$$

The dolphin is out of the water for 1 second.

2. When does the dolphin reach its greatest height?

t-coordinate of vertex $= ?$

$$t = \frac{0+1}{2} = 0.5$$

The dolphin reaches its greatest height at 0.5 seconds.

3. What is the dolphin's height after 1.5 seconds?

$$h(1.5) = ?$$

$$h(1.5) = -16(1.5)(1.5-1) = -12$$

The negative result means the dolphin has no height. It has already returned to the water.