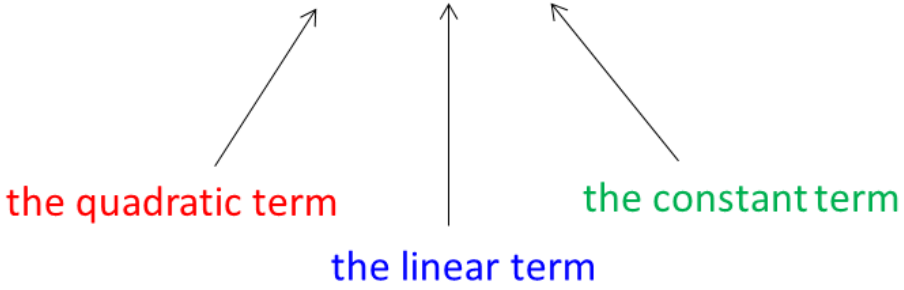
Objective: Solve Quadratic Equations using the Quadratic Formula

Concept

The terms of quadratic functions and quadratic equations have names that are based on the power of the variable in that term.

Standard Form of a Quadratic Equation

$$ax^2 + bx + c = 0$$



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Concept

All quadratic equations can be solved using what is called the Quadratic Formula. However, in most problems, the Quadratic Formula is best used when no other strategy, such as factoring or the square root property, is possible.

Given the quadratic equation $ax^2 + bx + c = 0$, the solutions can be found using what is called the Quadratic Formula.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$$

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Concept

Steps to Solve a Quadratic Equation Using the Quadratic Formula

1. Write the equation in standard form: $ax^2 + bx + c = 0$
2. Identify the values of a, b, and c.
3. Substitute the values into the Quadratic Formula.
4. Calculate and simplify the solutions.

Objective: Solve Quadratic Equations using the Quadratic Formula

Ex) Solve using the Quadratic Formula. Give exact solutions in simplest form. State whether the solutions are rational, irrational, or imaginary.

$$3x^2 + 1 = 6x$$

$-6x \quad -6x$

① $3x^2 - 6x + 1 = 0$ standard form

② $a = 3 \quad b = -6 \quad c = 1$

③ $x = \dfrac{-1(b) \pm \sqrt{(b)^2 - 4ac}}{2a}$

$x = \dfrac{-1(-6) \pm \sqrt{(-6)^2 - 4(3)(1)}}{2(3)}$

④ $x = \dfrac{6 \pm \sqrt{36 - [4(3)(1)]}}{6}$

$x = \dfrac{6 \pm \sqrt{36 - 12}}{6}$

$x = \dfrac{6 \pm \sqrt{24}}{6} = \dfrac{6 \pm 2\sqrt{6}}{6}$ ($\sqrt{4}\cdot\sqrt{6}$)

$= \dfrac{6}{6} \pm \dfrac{2\sqrt{6}}{6}$ ($6 = 3$)

$= 1 \pm \dfrac{\sqrt{6}}{3}$

solutions: $1 - \frac{\sqrt{6}}{3}, \; 1 + \frac{\sqrt{6}}{3}$

irrational

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Ex) Solve using the Quadratic Formula. Give exact solutions in simplest form. State whether the solutions are rational, irrational, or imaginary.

$$8x^2 + 14x = 15$$

$-15 \quad -15$

① $8x^2 + 14x - 15 = 0$

② $a = 8 \quad b = 14 \quad c = -15$

③ $x = \dfrac{-1(b) \pm \sqrt{(b)^2 - 4ac}}{2a}$

④ $x = \dfrac{-1(14) \pm \sqrt{(14)^2 - [4(8)(-15)]}}{2(8)}$

$-60 \times 8 = -480$

$x = \dfrac{-14 \pm \sqrt{196 - (-480)}}{16}$ $\quad 196 + 480 = 676$

$14 \times 14 = 56 + 140 = 196$

$= \dfrac{-14 \pm \sqrt{676}}{16}$

$26 \times 26 = 156 + 520 = 676$

$x = \dfrac{-14 \pm 26}{16} \rightarrow \dfrac{-14 + 26}{16}, \dfrac{-14 + 26}{16}$

reduce $\dfrac{-40}{16} \quad \dfrac{12}{16}$

$\dfrac{-5}{2} \quad \dfrac{3}{4}$

solutions: $\dfrac{-5}{2}, \dfrac{3}{4}$

rational

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Ex) Solve using the Quadratic Formula. Give exact solutions in simplest form. State whether the solutions are rational, irrational, or imaginary.

$$x^2 = 2x - 11$$

① $-2x \quad -2x \quad +11$

$+11$

$1x^2 - 2x + 11 = 0$

② $a = 1 \quad b = -2 \quad c = 11$

③ $$x = \frac{-1(b) \pm \sqrt{(b)^2 - 4ac}}{2a}$$

$$x = \frac{-1(-2) \pm \sqrt{(-2)^2 - [4(1)(11)]}}{2(1)}$$

④ $$x = \frac{2 \pm \sqrt{4 + -44}}{2}$$

$$x = \frac{2 \pm \sqrt{-40}}{2}$$

$\sqrt{-40}$

$\sqrt{40} \cdot \sqrt{-1}$

$\sqrt{4}\sqrt{10} \cdot (-1)$

$2\sqrt{10}\, i$

$$x = \frac{2 \pm 2\sqrt{10}\, i}{2}$$

$$x = \frac{2}{2} \pm \frac{2\sqrt{10}}{2}\, i$$

$$= 1 \pm \sqrt{10}\, i$$

solutions: $1 - \sqrt{10}\, i,\ 1 + \sqrt{10}\, i$

imaginary

Objective: Solve Quadratic Equations using the Quadratic Formula

Closure

Matthew and Gerald are both solving the same quadratic equation using the Quadratic Formula. One of them set the problem up incorrectly. Identify who set up the problem incorrectly and identify all mistakes made.

$$2x^2 - 8x = 5$$

Matthew's Work

$$x = \frac{-1(-8) \pm \sqrt{(2)^2 - 4(2)(5)}}{2}$$

Gerald's Work

$$x = \frac{-1(-8) \pm \sqrt{(-8)^2 - 4(2)(-5)}}{2(2)}$$

Matthew set up the problem incorrectly. He made three mistakes.

1. He used a instead of b and wrote $(2)^2$ instead of $(-8)^2$.
2. He used the wrong value for c. Standard form is $2x^2 - 8x - 5 = 0$, so $c = -5$.
3. He forgot to multiply the 2 by a in the denominator. The denominator should be $2(2)$.