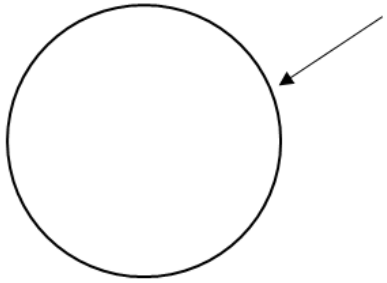
Objective: Find the volume of a sphere in context.

Prior Knowledge

Find the radius of the circle.



Circumference = 24π *in*

$C = \pi \cdot d$

$C = \pi \cdot 2 \cdot r$

$C = 2\pi r$

$\frac{24\pi}{\pi} = \frac{2\pi r}{\pi}$

$\frac{24}{2} = \frac{2r}{2}$

$r = 12$ in

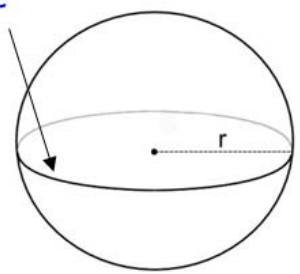
The radius is 12 inches.

Objective: Find the volume of a sphere in context.

Concept

The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere.

great circle



Note: **Great circles** are the largest circles that lie on a sphere and **have the same radius and center as the sphere**.

Real World Connection: The shortest path between two points on a sphere lies along the great circle that passes through the points. For this reason, ships and airplanes generally follow routes that are arcs of great circles.

What would be the formula to calculate the volume of a hemisphere (half of a sphere)?

$$V = \frac{1}{2} \cdot \frac{4}{3}\pi r^3 \rightarrow V = \frac{2}{3}\pi r^3$$

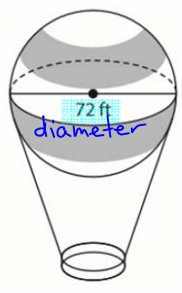
Objective: Find the volume of a sphere in context.

Ex) The figure represents a spherical helium-filled balloon. This tourist attraction allows up to 28 passengers at a time to ride in a gondola suspended underneath the balloon, as it cruises at an altitude of 500 ft. How much helium, to the nearest hundred gallons, does the balloon hold? Note: $1\ gal \approx 0.1337\ ft^3$

① find volume in ft^3

$V = \frac{4}{3}\pi r^3$

$r = \frac{72ft}{2} = 36ft$



$V = \frac{4}{3}\pi (36ft)^3$

calc. 4 ÷ 3 × π × 36 ^ 3 =

$V = 195,432.1958\ldots ft^3$

② convert ft^3 to gallons

$\frac{195,432.1958\ldots ft^3}{1} \cdot \frac{1\ gal}{0.1337\ ft^3}$

$\approx 1,461,721.734$ gallons

$\approx 1,461,700$ gallons

③ The balloon holds about 1,461,700 gallons of helium.

Objective: Find the volume of a sphere in context.

Practice) A spherical water tank has a diameter of 27 meters. How much water can the tank hold, to the nearest liter? Note: $1000\ L = 1\ m^3$

1. Find the radius of the tank.

$$radius = \frac{diameter}{2} = \frac{27m}{2} = 13.5\,m$$

2. Find the volume of the tank in cubic meters.

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\cdot\pi\cdot(13.5\,m)^3 \approx 10{,}305.9947\,m^3$$

3. Convert m^3 to liters. (round to the nearest liter)

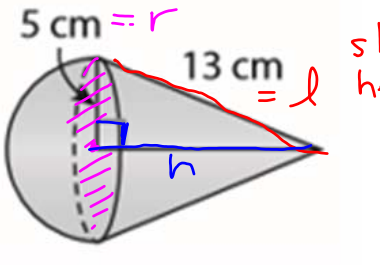
$$10{,}305.9947\,m^3 \cdot \frac{1000\,L}{1\,m^3} \approx 10{,}305{,}995\,liters$$

The tank can hold about 10,305,995 liters of water.

Objective: Find the volume of a sphere in context.

Ex) Find the volume of the composite figure. Round to the nearest cubic centimeter.

* one's place



① $V_{hemisphere} + V_{cone}$

$\frac{2}{3} \cdot \pi \cdot r^3 + \frac{1}{3} B h$

$\frac{2}{3} \cdot \pi \cdot r^3 + \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$

② find height h

$h^2 + r^2 = l^2$

$h^2 + 5^2 = 13^2$

$h^2 + 25 = 169$

$-25 \quad -25$

$h^2 = 144$

$\sqrt{h^2} = \sqrt{144}$

$h = 12$ cm

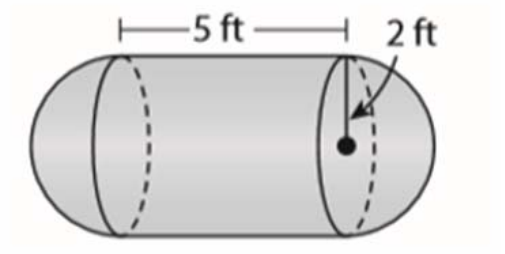
③ $\frac{2}{3} \cdot \pi \cdot (5\text{ cm})^3 + \frac{1}{3} \pi (5\text{ cm})^2 \cdot 12\text{cm}$

$\approx 576\ cm^3$

④ The volume of the composite figure is about $576\ cm^3$.

Objective: Find the volume of a sphere in context.

Practice) Find the volume of the composite figure. Round to the nearest tenth of a cubic foot.



1. Find the volume of the two hemispheres (which make a sphere).

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\cdot\pi\cdot(2\ ft)^3 \approx 33.5\ ft^3$$

2. Find the volume of the cylinder.

$$V = \pi r^2 h = \pi\cdot(2\ ft)^2\cdot 5\ ft \approx 62.8\ ft^3$$

3. Add the volumes.

$$33.5\ ft^3 + 62.8\ ft^3 \approx 96.3\ ft^3$$

The volume of the composite figure is about 96.3 cubic feet.

Objective: Find the volume of a sphere in context.

Concept

A gumball is in the shape of a sphere, with a spherical hole in the center. How might you calculate the volume of the solid portion of the gumball? What measurements are needed?

I could subtract the volume of the spherical hole from the volume of the gumball. I would need to know the radius of both the gumball and the hole.

Objective: Find the volume of a sphere in context.

Practice) A gumball with a diameter of 3 cm is in the shape of a sphere, with a spherical hole in the center of diameter 1 cm. What is the volume of the solid portion of the gumball? Round to the nearest tenth of a cubic centimeter.

1. Find the volume of the gumball.

$$radius = \frac{diameter}{2} = \frac{3\,cm}{2} = 1.5\,cm$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\cdot\pi\cdot(1.5\,cm)^3 \approx 14.1\,cm^3$$

2. Find the volume of the hole.

$$radius = \frac{diameter}{2} = \frac{1\,cm}{2} = 0.5\,cm$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\cdot\pi\cdot(0.5\,cm)^3 \approx 0.5\,cm^3$$

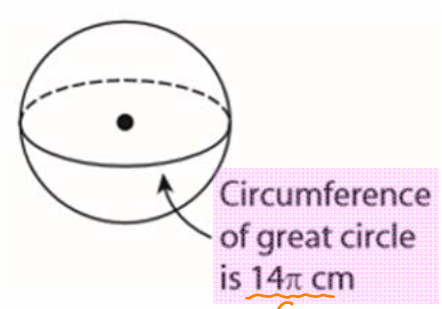
3. Subtract the volumes.

$$14.1\,cm^3 - 0.5\,cm^3 \approx 13.6\,cm^3$$

The volume of the solid portion of the gumball is about 13.6 cubic centimeters.

Objective: Find the volume of a sphere in context.

Ex) Find the volume of the sphere in cubic centimeters. Round to the nearest tenth.



① find radius

$C = 2 \cdot \pi \cdot r$

$\frac{14\pi}{\pi} = \frac{2 \cdot \pi \cdot r}{\pi}$

$14 = 2r$

$\frac{14}{2} = \frac{2r}{2}$

$r = 7\text{cm}$

② volume

$V = \frac{4}{3}\pi r^3$

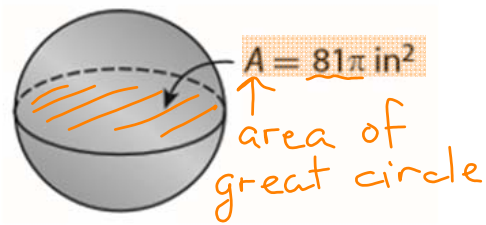
$V = \frac{4}{3}\pi (7\text{cm})^3$

$V \approx 1436.8\text{cm}^3$

③ The volume of the sphere is about 1436.8 cm^3.

Objective: Find the volume of a sphere in context.

Ex) Find the volume of the sphere in cubic inches. Round to the nearest tenth.



① find radius

$A = \pi r^2$

$\frac{81\pi}{\pi} = \frac{\pi r^2}{\pi}$

$81 = r^2$

$\sqrt{81} = \sqrt{r^2}$

$r = 9 \text{ in}$

② Volume

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (9 \text{ in})^3$

$\approx 3053.6 \text{ in}^3$

③ The volume of the sphere is about 3053.6 in^3.

Objective: Find the volume of a sphere in context.

Closure

Planet	Diameter (mi)
Mercury	3,032
Venus	7,521
Earth	7,926
Mars	4,222
Jupiter	88,846
Saturn	74,898
Uranus	31,763
Neptune	30,775

Margaret used the mathematics shown to find the volume of Saturn in terms of π.

$$V = \frac{4}{3}\pi r^2 = \frac{4}{3}\pi(74{,}898)^2$$
$$\approx \frac{4}{3}\pi(6{,}000{,}000{,}000)$$
$$\approx 8{,}000{,}000{,}000\pi\ mi^3$$

Explain the two errors Margaret made.

Margaret's two errors are that she used diameter instead of radius and she squared the radius when it should have been cubed.