Objective: Graph more complex rational functions.

## Concept

### Graphing Rational Functions of the Form $f(x) = \frac{p(x)}{q(x)}$

1. From the form $f(x) = \frac{p(x)}{q(x)}$, factor both numerator and denominator of the function, and reduce to find the simplest form of the function.
2. Find any holes.
3. Find all vertical and horizontal asymptotes.
4. Find all zeros, if they exist.
5. Find the $y$-intercept, if it exists.
6. Graph the asymptotes (as dashed lines), graph any intercepts, and graph enough other points to accurately sketch each piece of the curve.

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Ex) Graph the function.

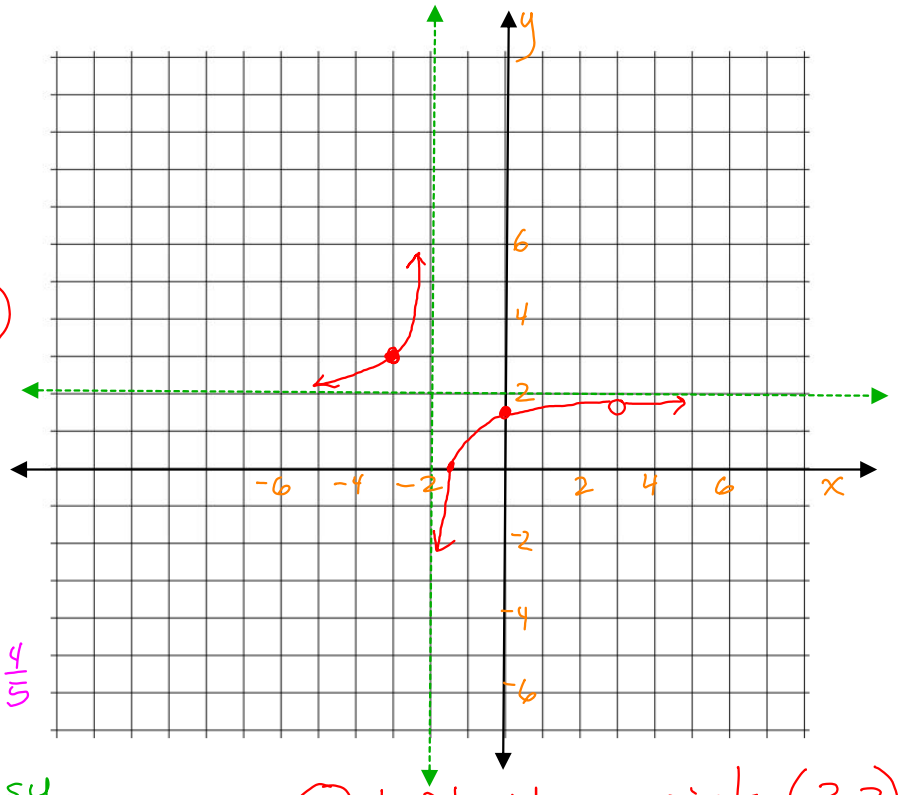
$$f(x) = \frac{2x^2 - 3x - 9}{x^2 - x - 6}$$

① $f(x) = \frac{(x-3)(2x+3)}{(x-3)(x+2)}$

$f(x) = \frac{2x+3}{x+2}$

② hole $(3, 1\frac{4}{5})$ (open circle)

$f(3) = \frac{6+3}{3+2} = \frac{9}{5} = 1\frac{4}{5}$




③ *Vert. asy. *horiz. asy.

$x+2=0$ $\quad \frac{2x}{x} = 2 \rightarrow *y=2$

*$x=-2$

④ left side point $(-3,3)$

let $x=-3$

$y = f(-3) = \frac{2(-3)+3}{-3+2} = \frac{-3}{-1} = 3$

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Ex) Graph the function.

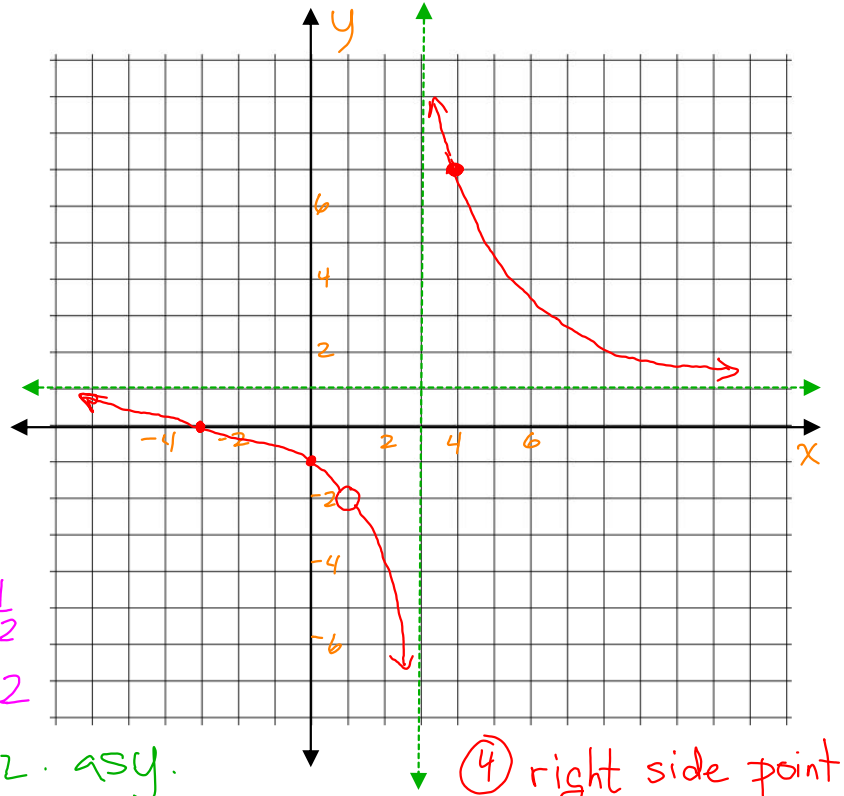
$$f(x) = \frac{x^2 + 2x - 3}{x^2 - 4x + 3}$$

① $f(x) = \frac{(x+3)(x-1)}{(x-3)(x-1)}$

$f(x) = \frac{x+3}{x-3}$

② hole (1, −2) (open circle)

$y = f(1) = \frac{1+3}{1-3} = \frac{4}{-2} = -2$




③ vert. asy $x = 3$ horiz. asy. $\frac{x}{x} = 1 \rightarrow y = 1$

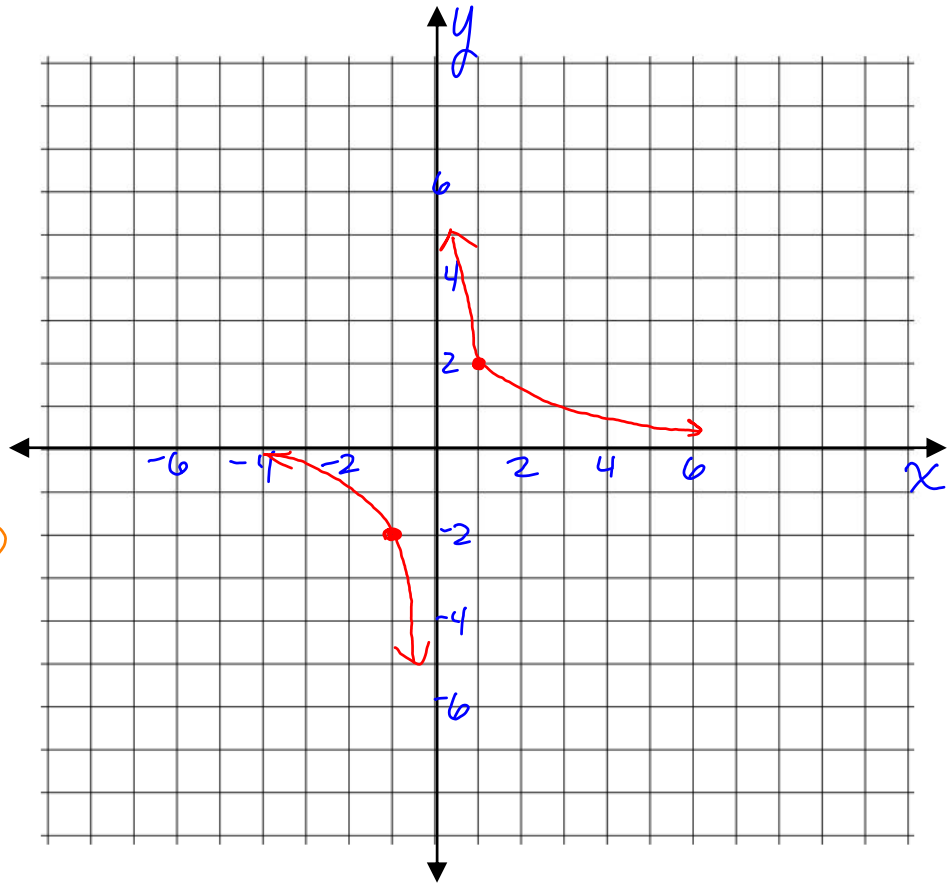
④ right side point let $x = 4$ (4, 7)

$y = f(4) = \frac{4+3}{4-3} = \frac{7}{1} = 7$

Objective: Graph more complex rational functions.

Ex) Graph the function.

$$f(x) = \frac{2}{x}$$ ← simplified function




① no hole

② vert. asy. $x = 0$ horiz. asy. $\frac{2}{x} \to y = 0$

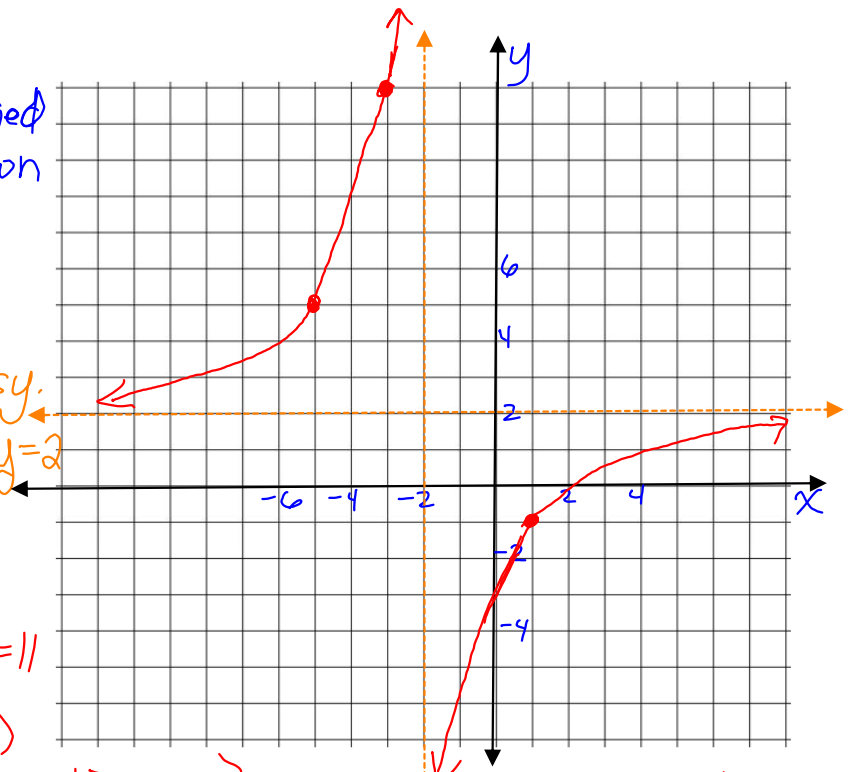
③ left side point
$x = -1$ $(-1, -2)$
$y = f(-1) = \frac{2}{-1} = -2$

right side point $(1, 2)$
let $x = 1$ $y = f(1) = \frac{2}{1} = 2$

Objective: Graph more complex rational functions.

Ex) Graph the function.

$$f(x) = \frac{2x-5}{x+2}$$ ← simplified function




① no hole

② vert. asy. horiz. asy.

$x = -2$ $\frac{2x}{x} = 2 \rightarrow y = 2$

③ left side point $(-3, 11)$

let $x = -3$

$y = f(-3) = \frac{-6-5}{-3+2} = \frac{-11}{-1} = 11$

let $x = -5$ $(-5, 5)$

$y = f(-5) = \frac{-10-5}{-5+2} = \frac{-15}{-3} = 5$

right side point $(1, -1)$

let $x = 1$

$y = f(1) = \frac{2-5}{1+2} = \frac{-3}{3} = -1$

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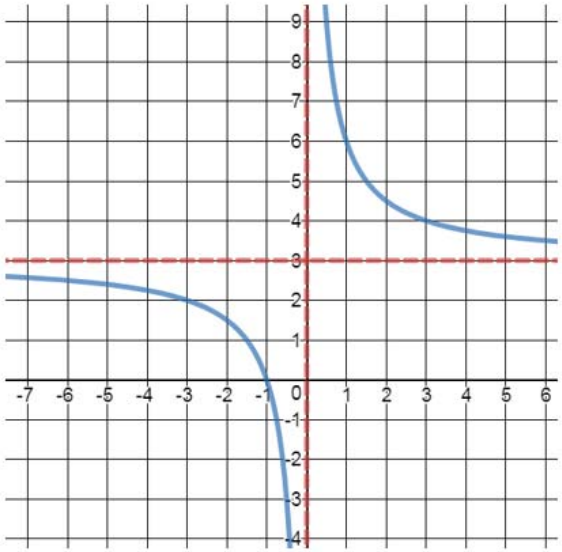
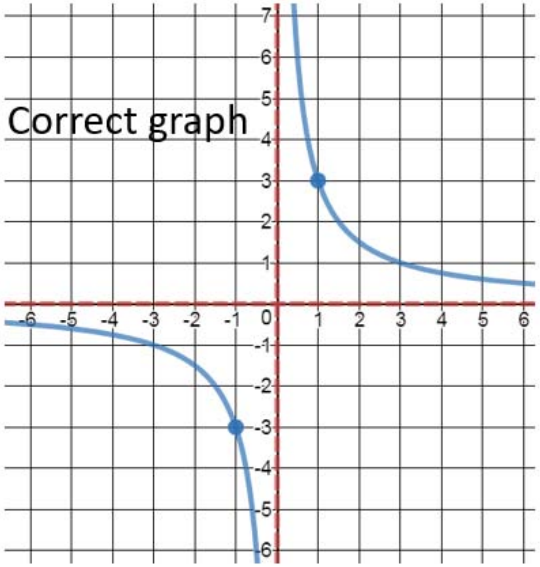
## Closure

Diana graphed the rational function shown. Her teacher said she made a mistake. What was Diana's mistake and how would you fix it?

$$f(x) = \frac{3}{x}$$






Diana's mistake is the horizontal asymptote (and also the points she used). I would change the horizontal asymptote to $y = 0$, and then graph the parts of the function using the points $(1,3)$ and $(-1,-3)$.