Objective: Simplify variable expressions with rational exponents.

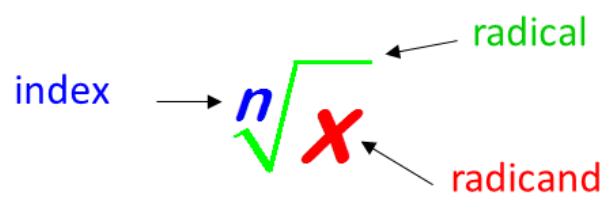
Concept

Rational and irrational numbers expressed in radical form can also be expressed with fractional exponents. When the number has a fractional exponent, it is said to be in rational exponent form.

$$\boxed{b^{\frac{1}{n}} = \sqrt[n]{b}}$$

and

$$\boxed{b^{\frac{p}{n}} = \sqrt[n]{b^p} \text{ or } b^{\frac{p}{n}} = \left(\sqrt[n]{b}\right)^p}$$



Objective: Simplify variable expressions with rational exponents.

Concept

Properties of Rational Exponents		
For all nonzero real numbers a and b and rational numbers m and n		
Words	**Numbers**	**Algebra**
Product of Powers Property: to multiply powers with the same base, add the exponents	$12^{\frac{1}{2}} \cdot 12^{\frac{3}{2}} = 12^{\frac{1}{2}+\frac{3}{2}} = 12^2 = 144$	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers Property: to divide powers with the same base, subtract the exponents	$\frac{125^{\frac{2}{3}}}{125^{\frac{1}{3}}} = 125^{\frac{2}{3}-\frac{1}{3}} = 125^{\frac{1}{3}} = 5$	$\frac{a^m}{a^n} = a^{m-n}$ or $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$
Power of a Power Property: to raise one power to another, multiply the exponents	$\left(8^{\frac{2}{3}}\right)^3 = 8^{\frac{2}{3}\cdot 3} = 8^2 = 64$	$(a^m)^n = a^{m \cdot n}$
Power of a Product Property: to find a power of a product, distribute the exponent	$(16 \cdot 25)^{\frac{1}{2}} = 16^{\frac{1}{2}} \cdot 25^{\frac{1}{2}} = 4 \cdot 5 = 20$	$(ab)^m = a^m b^m$
Power of a Quotient Property: to find a power of a quotient, distribute the exponent	$\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}} = \frac{2}{3}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Objective: Simplify variable expressions with rational exponents.

Concept

Properties of Rational Exponents

For all nonzero real numbers a and b and rational numbers m and n

Words	Numbers	Algebra
Negative Exponent Property: moving a power from numerator to denominator or vice versa changes the sign on the exponent	$36^{-\frac{1}{2}} = \frac{1}{36^{\frac{1}{2}}} = \frac{1}{6}$ $\frac{1}{36^{-\frac{1}{2}}} = \frac{36^{\frac{1}{2}}}{1} = \frac{6}{1} = 6$	$a^{-n} = \frac{1}{a^n}$ or $\frac{1}{a^{-n}} = a^n$
Zero Exponent Property: any monomial to a power of 0 is equal to 1	$(3)^0 = 1$	$(a)^0 = 1$

Objective: Simplify variable expressions with rational exponents.

Ex) Simplify the expression. Assume all variables are positive. Simplify numerical values as much as possible. Exponents in simplified form should be positive.

$$x^{\frac{6}{5}} \cdot x^{\frac{1}{2}}$$

product of powers

$$= x^{\frac{2 \cdot 6}{2 \cdot 5} + \frac{1 \cdot 5}{2 \cdot 5}}$$

$$= x^{\frac{12}{10} + \frac{5}{10}}$$

$$= \boxed{x^{\frac{17}{10}}}$$ simplified

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quotient of powers

$$\frac{y^{\frac{5}{8}}}{y^{\frac{3}{4}}}$$

$20 \quad \frac{5}{8} < \frac{3}{4} \quad 24$

$$= \frac{1}{y^{\frac{2\cdot 3}{2\cdot 4} - \frac{5}{8}}} = \frac{1}{y^{\frac{6}{8} - \frac{5}{8}}}$$

$$= \boxed{\frac{1}{y^{\frac{1}{8}}}}$$

simplified

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Ex) Simplify the expression. Assume all variables are positive. Simplify numerical values as much as possible. Exponents in simplified form should be positive.

$$\left(x^{\frac{3}{4}}\right)^{\frac{2}{3}}$$

power of a power

$$= x^{\frac{\cancel{3}^{1}}{\cancel{4}_{2}} \cdot \frac{\cancel{2}^{1}}{\cancel{3}_{1}}}$$

$$= \boxed{x^{\frac{1}{2}}}$$ simplified

Objective: Simplify variable expressions with rational exponents.

Ex) Simplify the expression. Assume all variables are positive. Simplify numerical values as much as possible. Exponents in simplified form should be positive.

$$\left(3x^{\frac{4}{3}}\right)\left(-2x^{-\frac{3}{2}}\right)$$

$$= 3 \cdot -2 \cdot x^{\frac{4}{3}} \cdot x^{\frac{-3}{2}}$$

$$= -6 \cdot x^{\frac{2 \cdot 4}{2 \cdot 3} + -\frac{3 \cdot 3}{2 \cdot 3}}$$

$$= -6 \cdot x^{\frac{8}{6} + \frac{-9}{6}}$$

$$= -6 \cdot x^{\frac{-1}{6}} = \boxed{\frac{-6}{x^{1/6}}}$$

simplified

Objective: Simplify variable expressions with rational exponents.

Ex) Simplify the expression. Assume all variables are positive. Simplify numerical values as much as possible. Exponents in simplified form should be positive.

① simplify inside parentheses

$$\left(\frac{\overset{8}{\cancel{24}}y^{\frac{6}{5}}}{\underset{1}{\cancel{3}}y^{\frac{3}{5}}}\right)^{\frac{10}{3}}$$

$$= \left(8y^{\frac{6}{5}-\frac{3}{5}}\right)^{10/3} = \left(8y^{\frac{3}{5}}\right)^{10/3}$$

② power of a product

$(ab)^n = a^n b^n$

$$= (8)^{\frac{10}{3}} \cdot \left(y^{\frac{3}{5}}\right)^{10/3}$$

$$= \left(\sqrt[3]{8}\right)^{10} \cdot y^{\frac{\cancel{3}^{1}}{\cancel{5}_{1}} \cdot \frac{\cancel{10}^{2}}{\cancel{3}_{1}}}$$

$$= 2^{10} \cdot y^2$$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$ ×2

$2^8 = 256$ ×2

$2^9 = 512$ ×2

$2^{10} = 1024$

$$= \boxed{1024y^2}$$ simplified

Objective: Simplify variable expressions with rational exponents.

Practice) Simplify the expression. Assume all variables are positive. Simplify numerical values as much as possible. Exponents in simplified form should be positive.

$$\left(\frac{\cancel{8}^{1}x^{\frac{8}{3}}}{\cancel{32}_{4}x^{\frac{1}{3}}}\right)^{\frac{3}{2}}$$

① simplify inside parentheses

$$= \left(\frac{x^{\frac{8}{3}}}{4x^{1/3}}\right)^{3/2} = \left(\frac{x^{\frac{8}{3}-\frac{1}{3}}}{4}\right)^{3/2}$$

$$= \left(\frac{x^{\frac{7}{3}}}{4}\right)^{3/2} = \frac{\left(x^{7/3}\right)^{3/2}}{(4)^{3/2}} = \frac{x^{\frac{7}{\cancel{3}}\cdot\frac{\cancel{3}^{1}}{2}}}{\left(\sqrt[2]{4}\right)^{3}}$$

② power of a quotient

$$\left(\frac{a}{b}\right)^{n} = \frac{a^{n}}{b^{n}}$$

simplified $= \boxed{\dfrac{x^{7/2}}{8}}$

Objective: Simplify variable expressions with rational exponents.

Ex) Simplify the expression. Assume all variables are positive. Simplify numerical values as much as possible. Exponents in simplified form should be positive.

$$\left(-5x^{\frac{7}{2}}\right)^2$$

power of a product

$(ab)^n$

$= a^n b^n$

$= (-5)^2 \cdot \left(x^{\frac{7}{2}}\right)^2$

$= 25 \cdot x^{\frac{7}{\cancel{2}_1} \cdot \frac{\cancel{2}}{1}^1}$

$= \boxed{25x^7}$ simplified

Objective: Simplify variable expressions with rational exponents.

Ex) Simplify the expression. Assume all variables are positive. Simplify numerical values as much as possible. Exponents in simplified form should be positive.

distribute $x^{\frac{1}{3}}\left(x^2-6\right)$

$= x^{\frac{1}{3}} \cdot x^{2} - 6 \cdot x^{\frac{1}{3}}$

$= x^{\frac{1}{3}+\frac{2 \cdot 3}{1 \cdot 3}} - 6x^{\frac{1}{3}}$

$= \boxed{x^{\frac{7}{3}} - 6x^{\frac{1}{3}}}$

simplified

Objective: Simplify variable expressions with rational exponents.

Ex) Rewrite the expression with a denominator of 1.

$$\frac{x^3+2}{\sqrt[2]{x^1}}$$

① rewrite $\sqrt{x}$ in exponent form

$$= \frac{x^3+2}{x^{\frac{1}{2}}}$$

② split

$$= \frac{x^3}{1\cdot x^{\frac{1}{2}}} + \frac{2}{1\cdot x^{\frac{1}{2}}}$$

③ negative exponent property

$$x^3 \cdot x^{-\frac{1}{2}} + 2\cdot x^{-\frac{1}{2}}$$

④ product of powers

$$x^{\frac{2}{2}\cdot\frac{3}{1} + -\frac{1}{2}} + 2x^{-\frac{1}{2}}$$

$$= \boxed{x^{\frac{5}{2}} + 2x^{-\frac{1}{2}}}$$

denominator of 1 form

Objective: Simplify variable expressions with rational exponents.

Closure

Darlene simplified a variable expression with rational exponents. Her work is shown. Identify the two mistakes she made. What is the correct answer?

simplify $3x^{\frac{7}{4}} \cdot 2x^{\frac{1}{2}}$

step 1: $5x^{\frac{7}{4}+\frac{1}{2}}$

step 2: $5x^{\frac{8}{6}}$

step 3: $\boxed{5x^{\frac{4}{3}}}$

In step 1 Darlene added the 3 and 2 instead of multiplying. In step 2 she added the fractions without first getting a common denominator.

simplify $3x^{\frac{7}{4}} \cdot 2x^{\frac{1}{2}}$

step 1: $3 \cdot 2 \cdot x^{\frac{7}{4}+\frac{1}{2}}$

step 2: $6 \cdot x^{\frac{7}{4}+\frac{2}{4}}$

step 3: $\boxed{6x^{\frac{9}{4}}}$