Objective: Solve logarithmic equations algebraically.

Concept

The **Properties of Logarithms** are valid for logarithms of all valid base values. This includes the common logarithm ($\log x$) and the natural logarithm ($\ln x$).

<table>
<tr><th colspan="2">Properties of Logarithms</th></tr>
<tr><td colspan="2">For any positive numbers $\boldsymbol{a, m, n, b}$ $(\boldsymbol{b \neq 1})$, $\boldsymbol{and\ c}$ $(\boldsymbol{c \neq 1})$, the following properties hold.</td></tr>
<tr><td>Power Property of Logarithms</td><td>$log_b m^n = n \cdot log_b m$</td></tr>
<tr><td>Product Property of Logarithms</td><td>$log_b m + log_b n = log_b (m \cdot n)$</td></tr>
<tr><td>Quotient Property of Logarithm</td><td>$log_b m - log_b n = log_b \left(\frac{m}{n}\right)$</td></tr>
<tr><td>Exponential Form</td><td>$\log_b x = n \ \rightarrow x = b^n$</td></tr>
<tr><td>Property of Equality for Logarithms</td><td>If $\log_b x = \log_b y$, then $x = y$.</td></tr>
</table>

Objective: Solve logarithmic equations algebraically.

Concept

Steps to Solve Logarithmic Equations that have Logarithms of the Same Base on Both Sides of the Equation

1. **Use the Properties of Logarithms** to write the equation with a single logarithm on each side.
2. **Use the Property of Equality for Logarithms** to set the arguments equal. **If $log_b\, x = log_b\, y$, then $x = y$.**
3. Solve this equation using algebra.
4. **Check for extraneous solutions.** (Recall, the argument of a logarithm must be positive. Therefore, solutions that would result in any argument in the original equation being negative or equal to zero are extraneous solutions and should be excluded from the solution set.)

Objective: Solve logarithmic equations algebraically.

Ex) Solve the equation.

$$\overset{\curvearrowright}{\textcircled{2}\ln x} = \overset{\curvearrowright}{\textcircled{6}\ln 2} - \overset{\curvearrowright}{\textcircled{4}\ln 2}$$

① power prop

$$\ln x^2 = \ln 2^6 - \ln 2^4$$

$$\ln x^2 = \ln 64 - \ln 16$$

quotient prop.

$$\ln x^2 = \ln \frac{64}{16}$$

$$\underline{\ln}\,\textcircled{x^2} = \underline{\ln}\,\textcircled{4}$$

② Prop. of equality

$$x^2 = 4$$

③

$$\sqrt{x^{\cancel{2}}} = \pm\sqrt{4}$$

$$x = \cancel{-2},\ 2 ✓$$

extraneous

④ check arguments >0

$$\boxed{\text{solution: } x = 2}$$

Objective: Solve logarithmic equations algebraically.

Ex) Solve the equation.

$$log_2(x^2 - 2) = log_2 10$$

① no properties needed

② prop. of equality

$$x^2 - 2 = 10$$

③

$$x^2 = 12$$

$$\sqrt{x^{\cancel{2}}} = \pm\sqrt{12}$$

$$x = -2\sqrt{3}\checkmark,\ 2\sqrt{3}\checkmark$$

④ check arguments > 0

$$\boxed{\text{solution: } x = -2\sqrt{3},\ 2\sqrt{3}}$$

Objective: Solve logarithmic equations algebraically.

Practice) Solve the equation.

$$ln\,80 - ln\,x^2 = ln\,2$$

$$\ln\frac{80}{x^2} = \ln 2$$

quotient property of logarithms

$$\frac{80}{x^2} = 2$$

equality property for logarithms

$$\frac{80}{x^2} = \frac{2}{1}$$

$$2x^2 = 80$$

$$x^2 = 40$$

$$\sqrt{x^2} = \pm\sqrt{40}$$

$x = -2\sqrt{10}, \quad x = 2\sqrt{10}$ *check*

$$\boxed{x = -2\sqrt{10}, \quad x = 2\sqrt{10}}$$

or

$$\ln 80 - \ln 2 = \ln x^2$$

$$\ln 40 = \ln x^2$$

$$40 = x^2$$

$$\sqrt{x^2} = \pm\sqrt{40}$$

$x = -2\sqrt{10}, \quad x = 2\sqrt{10}$ *check*

$$\boxed{x = -2\sqrt{10}, \quad x = 2\sqrt{10}}$$

Objective: Solve logarithmic equations algebraically.

Ex) Solve the equation.

$$log_6(x+4) + log_6(x-2) = 2log_6 2 + log_6 x$$

① power prop.

$$\log_6 (x+4) + \log_6 (x-2) = \log_6 4 + \log_6 x$$

product prop.

$$\log_6 \overset{(x+4)(x-2)}{(x^2+2x-8)} = \log_6 (4x)$$

② prop. of equality

$$x^2 + 2x - 8 = 4x$$

③

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$x - 4 = 0$ $x + 2 = 0$

$x = 4$ ✓ $\cancel{x = -2}$

extraneous

④ check. arguments > 0

$\boxed{\text{solution: } x = 4}$

Objective: Solve logarithmic equations algebraically.

Practice) Solve the equation.

$$ln(2x+1) + ln(x-3) = ln(x^2 - 3x)$$

$$\ln\left[(2x+1)(x-3)\right] = \ln\left(x^2 - 3x\right) \quad \text{product property of logarithms}$$

$$\ln\left[2x^2 - 5x - 3\right] = \ln\left(x^2 - 3x\right)$$

$$2x^2 - 5x - 3 = x^2 - 3x \quad \text{equality property for logarithms}$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x - 3 = 0 \ \ or \ \ x + 1 = 0$$

$$\cancel{x=3} \quad or \quad \cancel{x=-1} \quad check$$

$$\boxed{no\ solution} \quad or \quad \boxed{\varnothing}$$

Objective: Solve logarithmic equations algebraically.

Ex) Solve the equation.

$$log_3 2x \enclose{circle}{-} log_3(x-3) = log_3 5$$

① quotient prop.

$$\underline{\log_3} \enclose{circle}{\frac{2x}{x-3}} = \underline{\log_3} \enclose{circle}{5}$$

② prop. of equality

$$(\overset{1}{\cancel{x-3}}) \cdot \frac{2x}{\underset{1}{\cancel{x-3}}} = 5 \cdot (x-3)$$

③

$$2x = 5x - 15$$

$$-3x = -15$$

④ check.
arguments >0

$$x = 5 \checkmark$$

$$\boxed{\text{solution: } x = 5}$$

Objective: Solve logarithmic equations algebraically.

Practice) Solve the equation.

$$4ln2 - lnx = 3ln4$$

$\ln 2^4 - \ln x = \ln 4^3$ power property of logarithms

$\ln 16 - \ln x = \ln 64$

$\ln \frac{16}{x} = \ln 64$ quotient property of logarithms

$\frac{16}{x} = 64$ equality property for logarithms

$\frac{16}{x} = \frac{64}{1}$

$64x = 16$

$x = \frac{16}{64}$

$x = \frac{1}{4}$ *check*

$\boxed{x = \frac{1}{4}}$

Objective: Solve logarithmic equations algebraically.

Practice) Solve the equation.

$$log_8(x^2+4) - log_8 5 = log_8(2x-1)$$

$\log_8 \frac{x^2+4}{5} = \log_8 (2x-1)$ quotient property of logarithms

$\frac{x^2+4}{5} = 2x-1$ equality property for logarithms

$\frac{x^2+4}{5} = \frac{2x-1}{1}$

$x^2+4 = 5(2x-1)$

$x^2+4 = 10x-5$

$x^2-10x+9 = 0$

$(x-9)(x-1) = 0$

$x-9=0 \quad or \quad x-1=0$

$x=9 \quad or \quad x=1 \; check$

$\boxed{x=9 \quad or \quad x=1}$

Objective: Solve logarithmic equations algebraically.

Practice) Solve the equation.

$$2log x = log 2 + log(x + 12)$$

$\log x^2 = \log\left[2(x+12)\right]$ power property and product property of logarithms

$\log x^2 = \log\left[2x+24\right]$

$x^2 = 2x+24$ equality property for logarithms

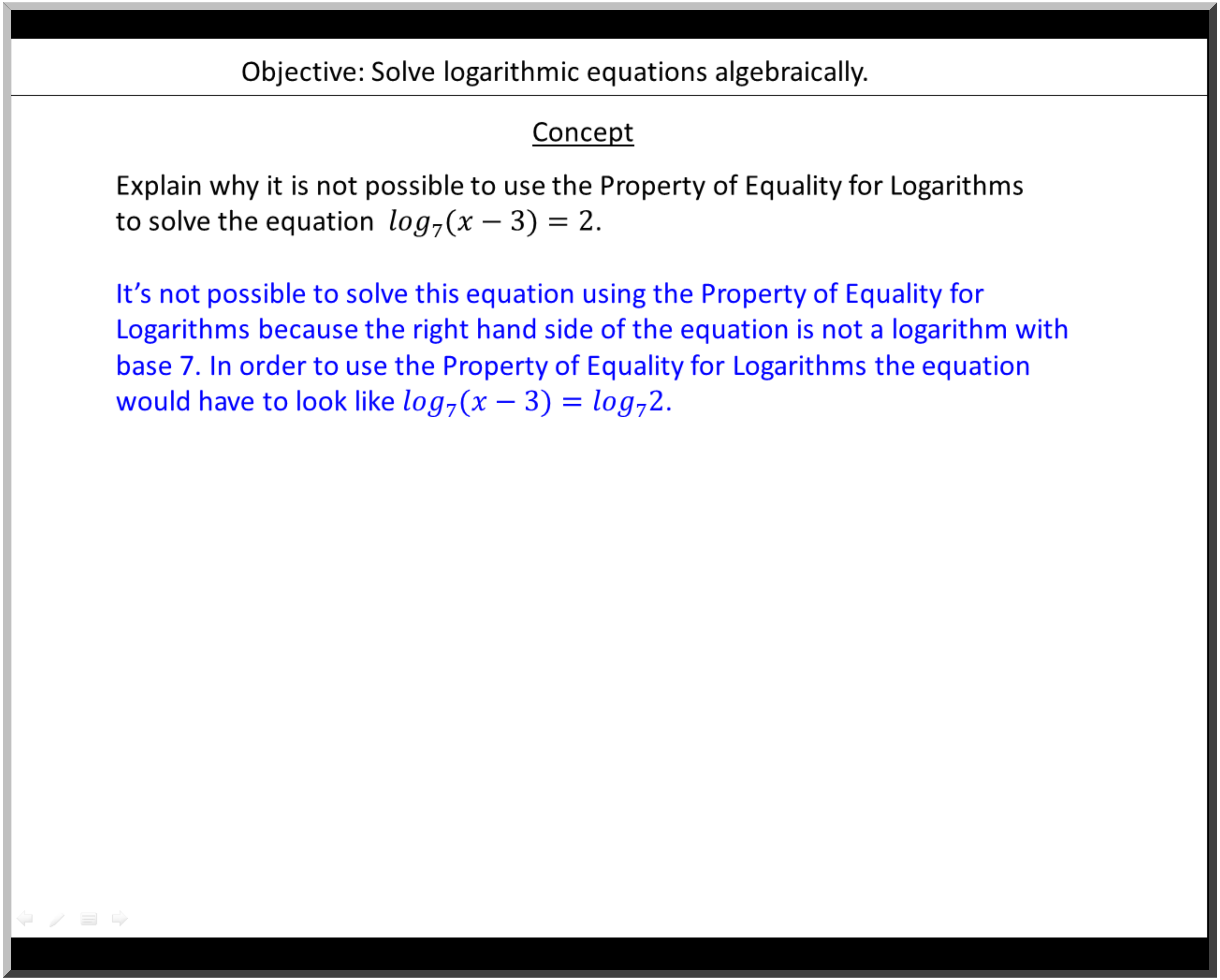
$x^2 - 2x - 24 = 0$

$(x-6)(x+4) = 0$

$x-6=0 \ \ or \ \ x+4=0$

$x=6 \quad or \quad \cancel{x=-4} \ \ check$

$\boxed{x=6}$



Objective: Solve logarithmic equations algebraically.

Concept

Explain why it is not possible to use the Property of Equality for Logarithms to solve the equation $log_7(x-3) = 2$.

It's not possible to solve this equation using the Property of Equality for Logarithms because the right hand side of the equation is not a logarithm with base 7. In order to use the Property of Equality for Logarithms the equation would have to look like $log_7(x-3) = log_7 2$.

Objective: Solve logarithmic equations algebraically.

Concept

Logarithmic equations of the form $log_b x = n$ cannot be solved using the Equality Property for Logarithms because the right side is not a logarithm of base b. **Notice, however, that the equation $log_b x = n$ is in Logarithmic Form.** Therefore, this equation **can be solved by writing it in the equivalent Exponential Form, $x = b^n$.**

Steps to Solve Logarithmic Equations that have a Logarithm on One Side of the Equation

1. **Use the Properties of Logarithms** to write the equation with a single logarithm on one side and a numeric value on the other.
2. Write the equation in **Exponential Form**. $log_b x = n \rightarrow x = b^n$
3. Solve this equation using algebra.
4. **Check for extraneous solutions.** (Recall, the argument of a logarithm must be positive. Therefore, solutions that would result in any argument in the original equation being negative or equal to zero are extraneous solutions and should be excluded from the solution set.)

Objective: Solve logarithmic equations algebraically.

Ex) Solve the equation.

$$log_2(x+5) - log_2 6 = 3$$

① quotient prop.

$$\underset{\text{base}}{\log_2} \overset{\text{argument}}{\frac{x+5}{6}} = \overset{\text{exp.}}{3} \leftarrow$$

logarithmic form

② exponential form

$$2^3 = \frac{x+5}{6}$$

③

$$6 \cdot 8 = \frac{x+5}{\cancel{6}_1} \cdot \cancel{6}^1$$

$$48 = x + 5$$

④ check.
arguments >0

$x = 43$ ✓

$$\boxed{\text{solution: } x = 43}$$

Objective: Solve logarithmic equations algebraically.

Ex) Solve the equation.

$$log(3x+1) = 2 \leftarrow$$ logarithmic form

① no prop. needed

$$\log_{\underset{base}{10}}(3x+1) = \overset{exp.}{2}$$

② exponential form $\rightarrow$ $10^2 = 3x+1$

③ $100 = 3x+1$

$99 = 3x$

$x = 33$ ✓

④ check. arguments >0

$$\boxed{\text{solution: } x = 33}$$

Objective: Solve logarithmic equations algebraically.

Practice) Solve the equation.

$$log_9(x+7) + log_9 3 = 2$$

$\log_9\left[3(x+7)\right] = 2$ product property of logarithms

$\log_9\left[3x+21\right] = 2$

$9^2 = 3x+21$ exponential form

$81 = 3x+21$

$60 = 3x$

$x = 20$ *check*

$\boxed{x=20}$

Objective: Solve logarithmic equations algebraically.

Ex) Solve the equation.

$$ln(2x-1) = 6 \leftarrow \text{logarithmic form}$$

① no prop. needed

$$\underset{\text{base}}{\overset{\downarrow}{\log_e}}(2x-1) = \overset{\text{exp.}}{6}$$

② exponential form

$$e^6 = 2x - 1$$

$$+1 \qquad +1$$

③

$$\frac{e^6+1}{2} = \frac{2x}{2}$$

④ check. arguments >0

$$x = \frac{e^6+1}{2} \quad ✓$$

$$\boxed{\text{solution: } x = \frac{e^6+1}{2}}$$

Objective: Solve logarithmic equations algebraically.

Practice) Solve the equation.

$$\ln x + \ln 9 = 4$$

$\ln(9x) = 4$ product property of logarithms

$e^4 = 9x$ exponential form

$x = \frac{e^4}{9}$ *check*

$$\boxed{x = \frac{e^4}{9}}$$

Objective: Solve logarithmic equations algebraically.

Practice) Solve the equation.

$$\log x + \log(x - 9) = 1$$

$$\log\left[x(x-9)\right] = 1$$ product property of logarithms

$$\log\left[x^2 - 9x\right] = 1$$

$$10^1 = x^2 - 9x$$ exponential form

$$10 = x^2 - 9x$$

$$x^2 - 9x - 10 = 0$$

$$(x-10)(x+1) = 0$$

$$x - 10 = 0 \quad or \quad x + 1 = 0$$

$$x = 10 \quad or \quad \cancel{x = -1} \quad check$$

$$\boxed{x = 10}$$

Objective: Solve logarithmic equations algebraically.

Closure

Explain when you must use the Property of Equality for Logarithms and when you must use Exponential Form to solve a logarithmic equation.

You must **use the Property of Equality for Logarithms** when the equation is written with a single logarithm of the same base on both sides.

You must **use Exponential Form** when only one side of the equation is a logarithm and the other is a numeric value.