Objective: Solve triangle problems in context.

Concept

Solving Triangle Problems in Context

1. Draw and label a triangle with the given information.

   A. **If the triangle is a right triangle. Use the Pythagorean Theorem and the definitions of sine, cosine, and tangent**.

   B. **If the triangle is a non-right triangle.** Determine whether the Law of Sines or the Law of Cosines must be used. If the Law of Sines must be used, determine if there is one triangle or two triangles.

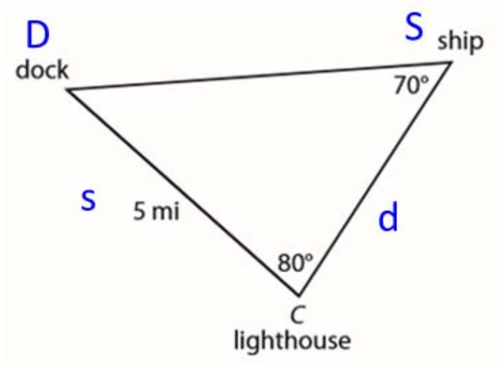
2. Interpret the solution in terms of the context and write your conclusion.

Objective: Solve triangle problems in context.

A ship has been damaged in a storm and is trying to reach safe harbor before sinking. The ship is between a lighthouse and a dock, forming a 70° angle between them. From the lighthouse, an 80° angle is formed between the dock and the ship.

a) If the lighthouse is 5 miles from the dock, how far, to the nearest tenth of a mile, is the ship from the lighthouse?

step 1:



step 2: Use Law of Sines with AAS → one triangle exists

step 3

$$m\angle D = 180^\circ - (80^\circ + 70^\circ) = 30^\circ$$

$$\frac{\sin D}{d} = \frac{\sin S}{s}$$

$$\frac{\sin 30^\circ}{d} = \frac{\sin 70^\circ}{5}$$

$$d = \frac{5 \cdot \sin 30^\circ}{\sin 70^\circ} \approx 2.7\,mi$$

The ship is about 2.7 miles from the lighthouse.

b) If the ship travels at an average speed of 28 miles per hour, how long, to the nearest minute, will it take the ship to reach the lighthouse?

$$\frac{2.7\,miles}{1} \cdot \frac{1\,hour}{28\,miles} \approx 0.10\,hours \rightarrow \frac{0.10\,hours}{1} \cdot \frac{60\min}{1\,hour} \approx 6\min$$

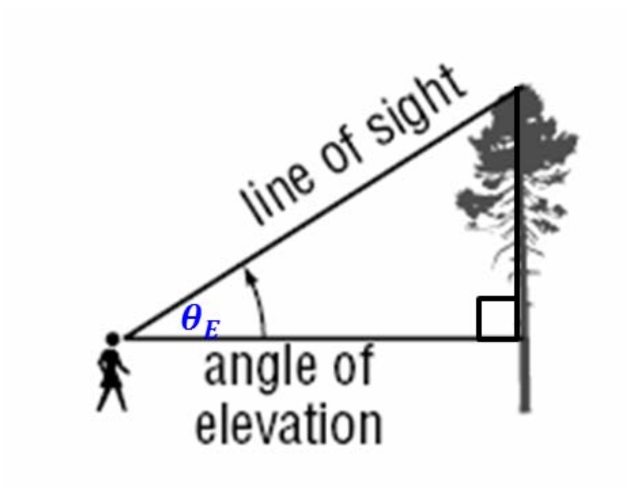
The ship will reach the lighthouse in about 6 minutes.

Objective: Solve triangle problems in context.

Concept

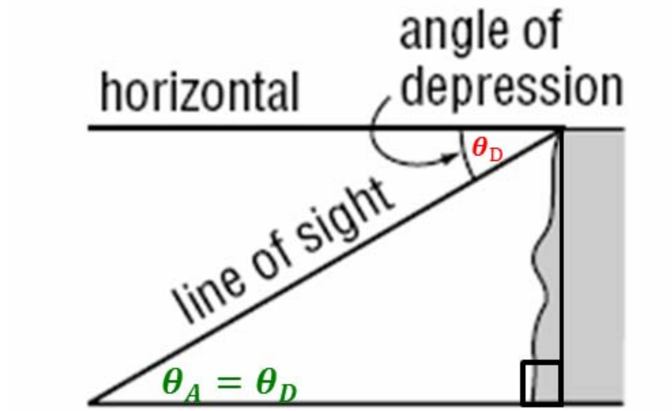
Angle of Elevation

The **angle of elevation, θ_E,** is the **angle between the horizontal and the line of site when looking upward**.



Angle of Depression

The **angle of depression, θ_D,** is the **angle between the horizontal and the line of site when looking downward**.

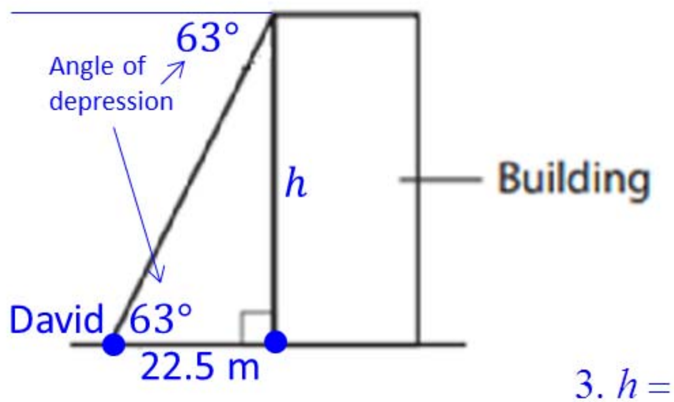


Because the line of sight is a transversal that intersects two parallel lines, alternate interior angles are congruent. This means angle θ_A is the same measure as the angle of depression θ_D.

Objective: Solve triangle problems in context.

David stands at the base of a building and jogs away from the building at 2.5 meters per second for 9 seconds. James is standing on the top of the building and measures an angle of depression of 63° from the top of the building to David. To the nearest foot, how tall is the building? Note: $1\ meter = 3.28084\ feet$

1.



2. $\tan 63^\circ = \dfrac{h}{22.5}$

$h = 22.5 \tan 63^\circ$

$h = 44.158...meters$

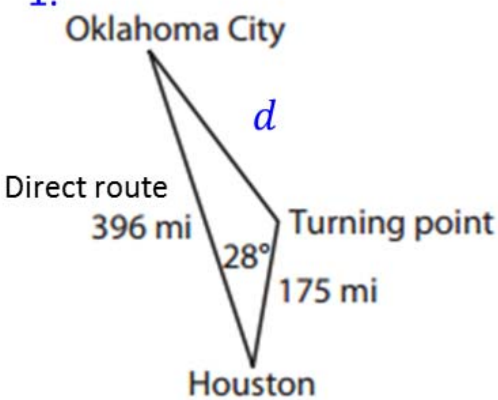
3. $h = \dfrac{44.158...meters}{1} \cdot \dfrac{3.28084\ feet}{1 meter} \approx 145\ feet$

The height of the building is approximately 145 feet.

Objective: Use the Law of Cosines to Solve Triangles

A pilot is flying from Houston to Oklahoma City. The direct route is 396 miles. To avoid a thunderstorm, the pilot flies *28°* off of the direct route for a distance of 175 miles. He then makes a turn and flies straight on to Oklahoma City. To the nearest mile, how much longer than the direct route was the route taken by the pilot?

1.



2. Find the distance, d, from the turning point to Oklahoma City using the Law of Cosines.

$$d^2 = 396^2 + 175^2 - 2(396)(175)\cos 28^\circ$$

$$d = \sqrt{396^2 + 175^2 - 2(396)(175)\cos 28^\circ}$$

$$d \approx 255\, miles$$

3. Find the difference between the distance flown and the direct route distance.

$$255\ mi + 175\ mi \approx 430\ mi$$

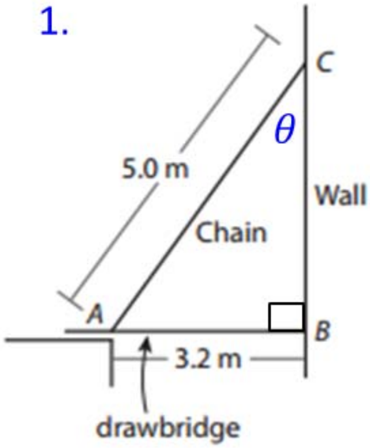
$$430\ mi - 396\ mi \approx 34\ mi$$

The route taken by the pilot was about 34 miles longer than the direct route.

Objective: Solving Right Triangle Problems in Context

A drawbridge at the entrance to an ancient castle is raised and lowered by a pair of chains. The chains are 5 meters long and are attached to the drawbridge 3.2 meters from the base. To the nearest degree, what is the angle the chain makes with the wall?

1.



2. $\sin\theta = \dfrac{3.2}{5}$

$$\theta = \sin^{-1}\left(\frac{3.2}{5}\right)$$

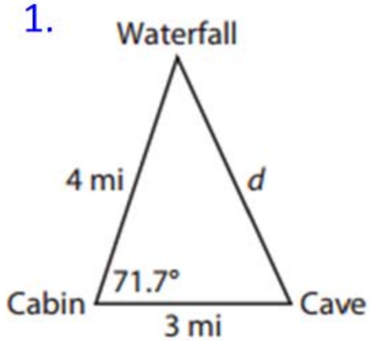
$$\theta \approx 40^\circ$$

The chain makes an angle of about 40° with the wall.

Objective: Use the Law of Cosines to Solve Triangles

Lucas is staying at a cabin and plans to go on a hike in the morning to a cave and then from there to a waterfall. He knows the hike from his cabin to the cave is 3 miles and from his cabin to the waterfall is 4 miles but he doesn't know the distance from the cave to the waterfall. From the cabin, an angle of 71.7° is formed between the cave and the waterfall. If Lucas hikes at an average of 2.5 miles per hour, estimate to the nearest tenth of an hour how long it will take him to reach the waterfall.

1.



2. Use the Law of Cosines to find d.

$$d^2 = 4^2 + 3^2 - 2(4)(3)\cos 71.7^\circ$$

$$d = \sqrt{4^2 + 3^2 - 2(4)(3)\cos 71.7^\circ}$$

$$d = 4.179\ldots miles$$

3. Determine the time for the hike from the cabin to the waterfall.
Total distance is 7.179… miles.

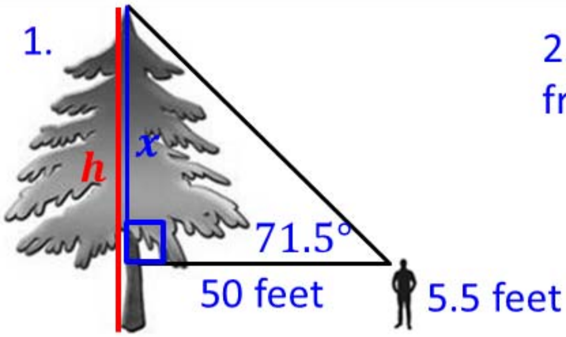
It will take Lucas about 2.9 hours to reach the waterfall.

$$\frac{7.179\ldots\ miles}{1} \cdot \frac{1\,hour}{2.5\,miles} \approx 2.9\,hours$$

Objective: Solving Right Triangle Problems in Context

A 5.5-foot tall person standing 50 feet from the base of a tree sights an angle of elevation from his line of sight to the top of the tree at 71.5°. Estimate the height of the tree in feet and inches. Round to the nearest inch.

1.



2. Let x be the part of the height of the tree from the line of sight to the top.

$$\tan 71.5^\circ = \frac{x}{50}$$

$$x = 50 \cdot \tan 71.5^\circ$$

$$x = 149.434\ldots \, feet$$

3. The total height, h, of the tree is the sum of x and the height of the person.

$$h = 149.434 + 5.5 = 154.934\ldots \, feet$$

4. $h = 154 \; feet \;\; 11 inches$

$$\frac{0.934\ldots \, feet}{1} \cdot \frac{12 \, inches}{1 \, foot} \approx 11 inches$$

The height of the tree is about 154 feet 11 inches.