Objective: Create Logarithmic Models from Exponential Models

Relevance

There are standard formulas that involve logarithms, such as the formula for measuring the loudness of sounds and the magnitude of an earthquake. It's also possible to develop logarithmic models from exponential models.

Steps to Change an Exponential Model to a Logarithmic Model

1. Simplify any function notation to a single variable.
2. Use algebra to isolate the power expression.
3. Write the equation in logarithmic form.
4. Solve for the independent variable and simplify, if necessary, to create a function that is as easy to use as possible.

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Ex) The mass in milligrams of beryllium-11, a radioactive isotope, in a 1200-milligram sample at time t in seconds is given by the function $m(t) = 1200e^{-0.05t}$.

a) Find a logarithmic model that can be used to find time in seconds.

$m(t) = 1200e^{-0.05t}$

$\frac{m}{1200} = \frac{1200e^{-0.05t}}{1200}$

$\frac{m}{1200} = e^{-0.05t}$

$\log_e \frac{m}{1200} = -0.05t$

$\frac{1}{-0.05} \ln \frac{m}{1200} = \frac{-0.05t}{-0.05}$

$-20 \ln \frac{m}{1200} = t$

$t = -20 \ln \frac{m}{1200}$

or

$t(m) = -20 \ln \frac{m}{1200}$

b) When, to the nearest second, will there be 5 milligrams of beryllium-11 remaining?

$t = ?$ when $m = 5$ mg

Use $t = -20 \ln \frac{m}{1200}$

$t = -20 \ln \frac{5}{1200}$

≈ 110 sec

There will be 5 mg of beryllium-11 remaining after about 110 seconds.

c) How many milligrams of beryllium-11 will remain after 30 seconds? Round to three decimal places.

$m = ?$ when $t = 30$ sec

Use $m(t) = 1200e^{-.05t}$

$m = 1200e^{-.05(30)}$

≈ 267.756 mg

After 30 seconds, about 267.756 mg of beryllium-11 will remain.

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Practice) The radioactive isotope fluorine-18 is used in medicine to produce images of internal organs and detect cancer. The percentage of fluorine-18 remaining after h hours is modeled by the function $p(h) = 100e^{-0.378h}$.

a) Find a logarithmic model that can be used to estimate time in hours.

$$p = 100e^{-0..378h}$$

$$\frac{p}{100} = e^{-0.378h}$$

$$\log_e \frac{p}{100} = -0.378h$$

$$\ln \frac{p}{100} = -0.378h$$

$$\frac{1}{-0.378} \cdot \ln \frac{p}{100} = h$$

$$h(p) = \frac{-500}{189} \ln \frac{p}{100}$$

or

$$h(p) = -2.646 \ln \frac{p}{100}$$

b) What percent of the fluorine-18 remains after 6.7 hours? Round to the nearest tenth.

$$p(6.7) = 100e^{-0.378(6.7)}$$

$$\approx 7.9$$

After 6.7 hours, there will be about 7.9% of the fluorine-18 remaining.

c) When, in hours and minutes, will there be 5% of the fluorine-18 remaining? Round to the nearest minute.

$$h(5) = \frac{-500}{189} \ln \frac{5}{100}$$

$$t \approx 7.9252...hours \approx 7\,hours\ 56\,\text{min}$$

There will be 5% of the fluorine-18 remaining after about 7 hours 56 minutes.

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Ex) Ajay made a steaming pot of stew. When the temperature of the stew reached $70°\ C$, Ajay turned the stove off and the stew began cooling. The temperature, T (measured in degrees Celsius), of the stew, m minutes since Ajay turned the stove off can be modeled by the function $T(m) = 20 + 50(10)^{-0.04m}$.

a) Find a logarithmic model that can be used to find time in minutes.

$T(m) = 20 + 50(10)^{-.04m}$

$T = 20 + 50(10)^{-.04m}$

$-20 \quad -20$

$\frac{T-20}{50} = \frac{50(10)^{-.04m}}{50}$

$\frac{T-20}{50} = 10^{-.04m}$

$\log_{10} \frac{T-20}{50} = -.04m$

$\frac{1 \cdot \log \frac{T-20}{50}}{-.04} = \frac{-0.04m}{-.04}$

$-25 \log \frac{T-20}{50} = m$

$$\boxed{m = -25 \log \frac{T-20}{50} \quad \text{or} \quad m(T) = -25 \log \frac{T-20}{50}}$$

b) How long, to the nearest tenth of a minute, did it take the stew to reach a temperature of 30° C?

$m = ?$ when $T = 30°C$

use $m = -25 \log \frac{T-20}{50}$

$m = -25 \log \frac{30-20}{50}$

≈ 17.5 min

It took the stew about 17.5 minutes to reach a temperature of 30° C.

c) What is the temperature of the stew, to the nearest tenth of a degree, after 1 hour?

$T = ?$ when $m = 1$ hour $= 60$ min

use $T = 20 + 50(10)^{-.04m}$

$T = 20 + 50(10)^{-.04(60)}$

≈ 20.2 °C

After 1 hour, the temperature of the stew is about 20.2° C.

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Closure

The function $P = 500e^{0.005t}$ models an animal population, P, in a favorable environment after t years. The function $t = 200 \ln \frac{P}{500}$ models the time in years for the animal population to reach a certain amount.

David is solving the problem below. Explain to David how to decide which mathematical model he should use to solve the problem.

An animal population is being monitored by the national parks service. They would like to know what the animal population is expected to be in ten years. What would you tell them?

David should use the model $P = 500e^{0.005t}$ because the value of t is given as 10 years. Substituting 10 for t in this function and using a calculator will tell David what the expected animal population will be in ten years. He can then tell the national parks service what to expect.