Objective: Solve context problems using the Quadratic Formula

Concept

Given the quadratic equation $\boldsymbol{ax^2 + bx + c = 0}$, the solutions can be found using what is called the Quadratic Formula.

Quadratic Formula

$$\boldsymbol{x = \frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}}$$

Steps to Solve a Quadratic Equation Using the Quadratic Formula

1. Write the equation in standard form: $ax^2 + bx + c = 0$

2. Identify the values of a, b, and c.

3. Substitute the values into the Quadratic Formula.

4. Calculate and simplify the solutions.

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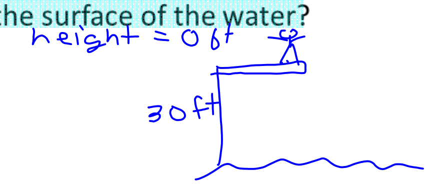
Solve using the quadratic formula. Round to the nearest hundredth if necessary. Write a sentence conclusion.

Ex) The quadratic function $h(t) = -16t^2 + vt + h$ models the height, in feet, of an object fired upward after t seconds, where v is the starting velocity and h is the starting height of the object.

A diver stands on a platform 30 feet above the surface of the water and jumps up and out with a beginning speed of 8 feet per second.

a) How long will it take for the diver to hit the surface of the water?



① write the function model.

$h(t) = -16t^2 + vt + h$

$h = 30$

$v = 8$

$h(t) = -16t^2 + 8t + 30$

② set up the problem

$t = ?$ when $h(t) = 0$

$0 = -16t^2 + 8t + 30$

$a = -16 \quad b = 8 \quad c = 30$

③ solve.

$$t = \frac{-1(b) \pm \sqrt{(b)^2 - 4ac}}{2a}$$

$$t = \frac{-1(8) \pm \sqrt{(8)^2 - 4(-16)(30)}}{2(-16)}$$

$$t = \frac{-8 \pm \sqrt{(8)^2 - 4(-16)(30)}}{-32}$$

$$t = \frac{-8 - \sqrt{(8)^2 - 4(-16)(30)}}{-32}$$

calc. $(-8 \pm \text{2nd}\sqrt{\ }((8)^2 - 4 \times -16 \times 30)) \div -32 =$

$t \approx 1.64$ sec ~~$t \approx -1.14$ sec~~

④ conclusion

The diver will hit the surface of the water after about 1.64 seconds.

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Ex) The quadratic function $h(t) = -16t^2 + vt + h$ models the height, in feet, of an object fired upward after t seconds, where v is the starting velocity and h is the starting height of the object.

A diver stands on a platform 30 feet above the surface of the water and jumps up and out with a beginning speed of 8 feet per second.

b) Can the diver reach a height of 35 feet above the water? If yes, when will the diver reach this height. If no, explain your reasoning.

h(t)

① model

$h(t) = -16t^2 + 8t + 30$

② set up the problem

$35 = -16t^2 + 8t + 30$

$-35 \qquad -35$

$0 = -16t^2 + 8t - 5$

a = -16, b = 8, c = -5

③ solve.

$t = \frac{-1(8) \pm \sqrt{(8)^2 - 4(-16)(-5)}}{2(-16)}$

$t = \frac{-8 \pm \sqrt{64 - 320}}{-32}$

$t = \frac{-8 \pm \sqrt{-256}}{-32}$ imaginary, domain error

calc. (-8 — 2nd √(-256)) ÷ -32 =

④ conclusion

The diver can't reach a height of 35 feet because the solutions are imaginary.

Objective: Solve context problems using the Quadratic Formula

Solve using the quadratic formula. Round to the nearest hundredth if necessary. Write a sentence conclusion.

Practice) The quadratic function $h(t) = -16t^2 + vt + h$ models the height, in feet, of an object fired upward after t seconds, where v is the starting velocity and h is the starting height of the object.

A football player released the ball 6 feet above the ground with an initial velocity of 18 feet per second.

a) If no other player touches the ball, when will it hit the ground?

$h(t) = -16t^2 + 18t + 6$

means $h(t) = 0$

$0 = -16t^2 + 18t + 6$

$a = -16, \ b = 18, \ c = 6$

$$t = \frac{-1(18) \pm \sqrt{(18)^2 - [4(-16)(6)]}}{2(-16)}$$

$$t = \frac{-18 \pm \sqrt{708}}{-32}$$

~~$t \approx -0.27$~~ or $t \approx 1.39$

The ball will hit the ground after about 1.39 seconds.

b) Can the ball reach a height of 10 feet? If yes, when will the ball reach this height. If no, explain your reasoning.

$h(t) = -16t^2 + 18t + 6$

means $h(t) = 10$

$10 = -16t^2 + 18t + 6$

$0 = -16t^2 + 18t - 4$

$a = -16, \ b = 18, \ c = -4$

$$t = \frac{-1(18) \pm \sqrt{(18)^2 - [4(-16)(-4)]}}{2(-16)}$$

$$t = \frac{-18 \pm \sqrt{68}}{-32}$$

$t \approx 0.30$ or $t \approx 0.82$

The ball will be at a height of 10 feet at about 0.30 seconds and again at about 0.82 seconds.

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Ex) The length and width of a patio are $(x + 9)$ feet and $(x + 7)$ feet, respectively. If the area of the patio is 190 square feet, what are the dimensions of the patio?

① write the model

patio is rectangular

Area = Length · Width

$190 = (x+9)(x+7)$

② solve

$190 = (x+9)(x+7)$

$190 = x^2 + 16x + 63$

$-190 \qquad -190$

$0 = 1x^2 + 16x - 127$

$a = 1 \quad b = 16 \quad c = -127$

$x = \frac{-1(16) \pm \sqrt{(16)^2 - 4(1)(-127)}}{2(1)}$

$x = \frac{-16 \pm \sqrt{(16)^2 - 4(1)(-127)}}{2}$

~~$x \approx -21.82$~~ makes length and width negative

$x \approx 5.82$

③ dimensions

length = (x+9) feet
= (5.82 + 9) feet
= 14.82 feet

width = (x + 7) feet
= (5.82 + 7) feet
= 12.82 feet

The dimensions of the patio are about 14.82 feet and 12.82 feet.

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Solve using the quadratic formula. Round to the nearest hundredth if necessary. Write a sentence conclusion.

Practice) The quarterback of a football team throws a pass to the team's receiver. The height h, in meters, of the football can be modeled by $h = -4.9t^2 + 3t + 1.75$, where t is the elapsed time in seconds. If the receiver catches the football at a height of 0.25 meters, how long does the ball remain in the air?

$h = -4.9t^2 + 3t + 1.75$

means $h(t) = 0.25$

$$0.25 = -4.9t^2 + 3t + 1.75$$

$$0 = -4.9t^2 + 3t + 1.5$$

$$a = -4.9,\ b = 3,\ c = 1.5$$

$$t = \frac{-1(3) \pm \sqrt{(3)^2 - [4(-4.9)(1.5)]}}{2(-4.9)}$$

$$t = \frac{-3 \pm \sqrt{38.4}}{-9.8}$$

$\require{cancel}\xcancel{t \approx -0.33}$ or $t \approx 0.94$

The ball remains in the air for about 0.94 seconds.

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Practice) A scientist is growing bacteria in a lab for study. One particular type of bacteria grows at a rate of $y = 2t^2 + 3t + 500$. A second bacteria grows at a rate of $y = 3t^2 + t + 300$. In both of these functions, y is the number of bacteria after t minutes. When is there an equal number of both types of bacteria?

given: $y = 2t^2 + 3t + 500$ and $y = 3t^2 + t + 300$

means $2t^2 + 3t + 500 = 3t^2 + t + 300$

$$0 = 1t^2 - 2t - 200$$

$$a = 1, \ b = -2, \ c = -200$$

$$t = \frac{-1(-2) \pm \sqrt{(-2)^2 - [4(1)(-200)]}}{2(1)}$$

$$t = \frac{2 \pm \sqrt{804}}{2}$$

~~$t \approx -13.18$~~ or $t \approx 15.18$

There are an equal number of both types of bacteria after about 15.18 minutes.