Objective: Use reasoning with three-dimensional shapes to solve modeling problems.

Concept

Strategies for Solving Non-Routine Problems

1. Draw a diagram, including units of measure.
2. Convert differing units of measure to the same units.
3. Break the problem into smaller steps.
4. Be aware of the final goal at each step.
5. Verify the units of measure correspond to the concept and context of the problem.

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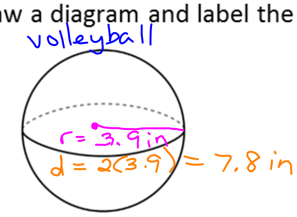
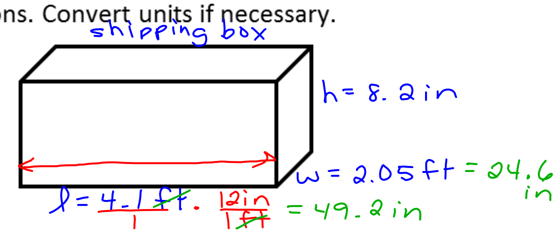
Ex) A volleyball company is considering shipping in bulk. The packing boxes that they would use to ship in bulk have a length of 4.1 feet, a width of 2.05 feet and a height of 8.2 inches. The volleyballs have a radius of 3.9 inches. If the volleyballs are stacked uniformly in the packing boxes, how many volleyballs would the company be able to ship in one box?

Step 1. Draw a diagram and label the dimensions. Convert units if necessary.





Step 2: Calculate the number of volleyballs that can fit within each dimension of the box.

$$\text{length} = \frac{\text{length box}}{\text{length v.b.}} = \frac{49.2\text{ in}}{7.8\text{ in}} = 6.307\ldots \rightarrow 6 \text{ volleyballs lengthwise}$$

$$\text{width} = \frac{\text{width box}}{\text{width v.b.}} = \frac{24.6\text{ in}}{7.8\text{ in}} = 3.153\ldots \rightarrow 3 \text{ volleyballs widthwise}$$

$$\text{height} = \frac{\text{height box}}{\text{height v.b.}} = \frac{8.2\text{ in}}{7.8\text{ in}} = 1.051\ldots \rightarrow 1 \text{ volleyball top to bottom}$$

Step 3: Calculate the total number of volleyballs that will fit in the box.

Multiply v.b. lengthwise × v.b. widthwise × v.b. heightwise

$= 6 \times 3 \times 1$

$= 18$ volleyballs

conclusion

The company can ship 18 volleyballs in one box.

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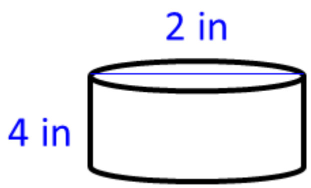
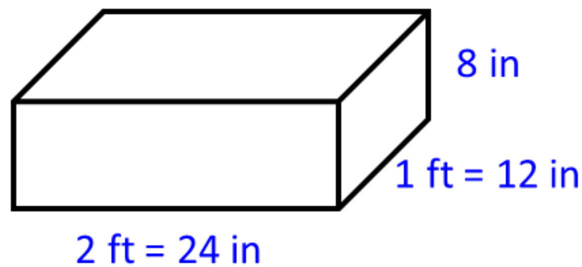
Practice) An olive company ordered new packing boxes to ship their cans of olives. The cans of olives are cylindrical and have a diameter of 2 inches and a height of 4 inches. The packing boxes have a length of 2 feet, a width of 1 foot and a height of 8 inches. How many cans of olives will the company be able to ship in one packing box?

Step 1. Draw a diagram and label the dimensions. Convert units if necessary.





Step 2: Calculate the number of cans that can fit within each dimension.

$$\frac{length\ of\ box}{diameter\ of\ can} = \frac{24\ in}{2\ in} = 12\ cans\ can\ fit\ the\ length$$

$$\frac{height\ of\ box}{height\ of\ can} = \frac{8\ in}{4\ in}$$
$$= 2\ cans\ can\ fit\ the\ height$$

$$\frac{width\ of\ box}{diameter\ of\ can} = \frac{12\ in}{2\ in} = 6\ cans\ can\ fit\ the\ width$$

Step 3: Calculate the total number of cans that will fit in the box.

$$total\ cans = (cans\ that\ fit\ length)(cans\ that\ fit\ width)(cans\ that\ fit\ height)$$
$$= (12)(6)(2) = 144$$

A total of 144 cans of olives can fit in the packing box.

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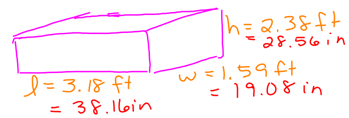
Ex) A basketball company is shipping basketballs in bulk to a wholesale store. The basketballs will be shipped in a container shaped like a rectangular prism. The container has a length of 3.18 feet, a width of 1.59 feet and a height of 2.38 feet. Each basketball has a radius of 4.75 inches. The company plans on filling the container with the maximum number of basketballs it can hold. Determine how much space is wasted inside the container.

① basketball sphere

rectangular prism (box)



② number of basketballs in each dimension

$\frac{\text{length box}}{\text{length b.b.}} = \frac{38.16\text{ in}}{9.5\text{ in}} = 4.016\ldots \rightarrow 4$ basketballs lengthwise

$\frac{\text{width box}}{\text{width b.b.}} = \frac{19.08\text{ in}}{9.5\text{ in}} = 2.008\ldots \rightarrow 2$ basketballs widthwise

$\frac{\text{height box}}{\text{height b.b.}} = \frac{28.56\text{ in}}{9.5\text{ in}} = 3.006\ldots \rightarrow 3$ basketballs heightwise

③ $4 \times 2 \times 3 = 24$ basketballs

④ Volume of all b.b.'s

$V = \frac{4}{3}\pi r^3 \times 24$

$V = \frac{4}{3}\pi (4.75\text{ in})^3 \times 24$

$\approx 10{,}774.1\text{ in}^3$

Volume of box

$V = Bh$

$V = l \cdot w \cdot h$

$V = (38.16)(19.08)(28.56)$

$\approx 20{,}794.3\text{ in}^3$

⑤ wasted space = Volume of box − Volume of all b.b.'s

$= 20{,}794.3\text{ in}^3 - 10{,}774.1\text{ in}^3$

$= 10{,}020.2\text{ in}^3$

conclusion

About 10,020.2 cubic inches of space is wasted inside the container.

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Closure

Explain why you use the diameter and not the radius of a sphere when determining how many spheres could fit within a given length.

The diameter is used because the diameter represents the length of a sphere at its widest point.