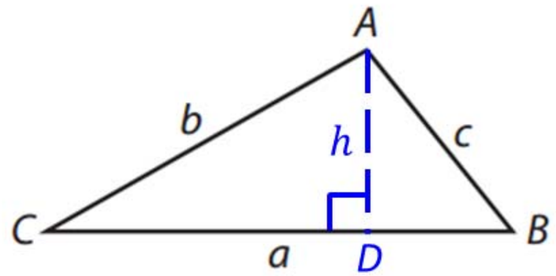
Objective: Find the Area of Non-Right Triangles

Concept

To use the area of a triangle formula $A = \frac{1}{2}bh$, both the base and height measurements of the triangle must be known. **For when the height is not known, a more general formula for the area of a triangle can be determined using trigonometry.**

1. Given non-right ΔABC, draw an altitude $\overline{AD}$ to represent the height, h, of the triangle.



The base of this triangle is now a, with height h.

2. Using right triangle ACD, write a trigonometric equation using sine and $\angle C$.

$$\sin C = \frac{h}{b}$$

3. Solve $\sin C = \frac{h}{b}$ for height, h.

$$h = b \sin C$$

4. Using the area of a triangle formula $A = \frac{1}{2}bh$, substitute a for the base b, and $bsinC$ for h.

$$A = \frac{1}{2}bh \rightarrow A = \frac{1}{2}ab \sin C$$

(Where a and b are two sides of a triangle and C is the included angle.)

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Concept

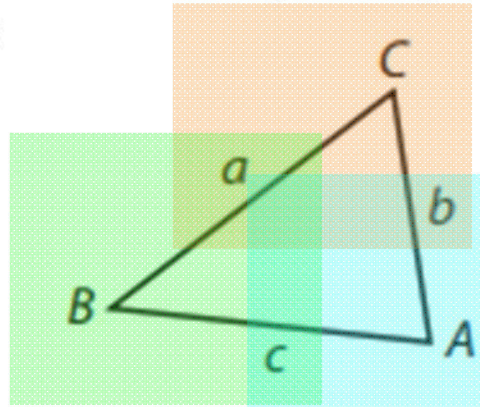
Area Formula for a Triangle Given Two Sides and the Included Angle

The area of ΔABC with sides $a, b, and\ c$ can be found using the lengths of two of its sides and the sine of the included angle:

$$A = \frac{1}{2} ab\ sinC$$

$$A = \frac{1}{2} ac\ sinB$$

$$A = \frac{1}{2} bc\ sinA$$



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Concept

Heron's Area Formula

Given any triangle with side lengths a, b, and c, the area of the triangle is

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where

$$s = \frac{a+b+c}{2}.$$ half the triangle's perimeter

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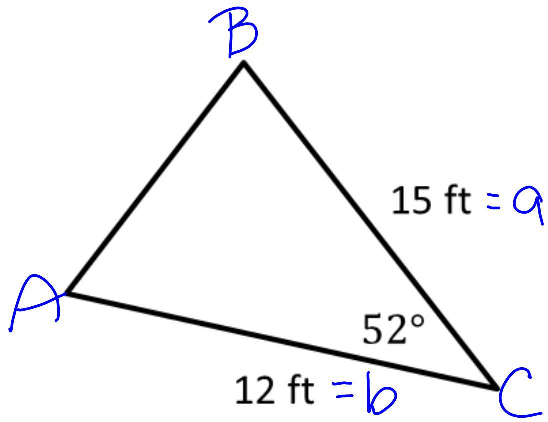
Steps to Find the Area of a Non-Right Triangle

1. Determine the given information.
   1. If two sides and the included angle are known, use $A = \frac{1}{2}ab\sin C$.
   2. If three sides are known, use Heron's Area Formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$.
2. If neither of these two is the case, find any needed measures using the Law of Sines and/or Law of Cosines and then find the area using the appropriate formula.

Objective: Find the Area of Non-Right Triangles

Ex) Find the area of the triangle to the nearest tenth.



① Use $A = \frac{1}{2} \cdot a \cdot b \cdot \sin C$

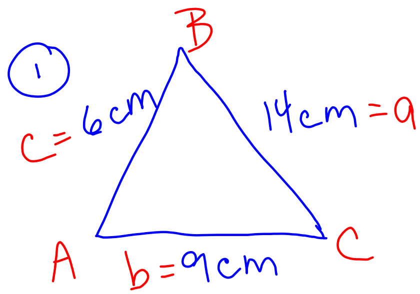
$A = \frac{1}{2}(15\,ft)(12\,ft)\sin 52^\circ$

$A \approx 70.9\ ft^2$

② The area of the triangle is about 70.9 square feet.

Objective: Find the Area of Non-Right Triangles

Ex) The sides of a triangle are 14 cm, 9 cm and 6 cm. Find the area of the triangle to the nearest tenth.



① Use Heron's Formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

② find s

$$s = \frac{a+b+c}{2} = \frac{14\text{ cm} + 9\text{ cm} + 6\text{ cm}}{2} \quad (29\text{cm})$$

$$s = 14.5\text{cm}$$

③ $$A = \sqrt{14.5\text{cm}(14.5\text{cm} - 14\text{cm})(14.5\text{cm} - 9\text{cm})(14.5\text{cm} - 6\text{cm})}$$

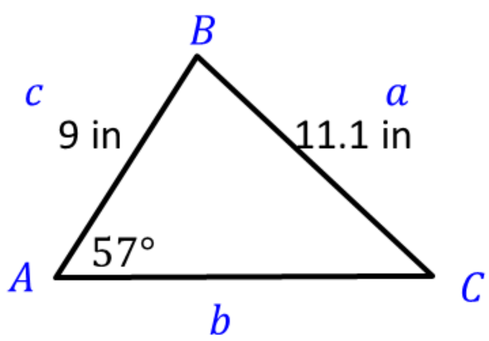
$$A = \sqrt{14.5\text{cm}(0.5\text{cm})(5.5\text{cm})(8.5\text{cm})}$$

$$A = \sqrt{338.9375\text{ cm}^4} \approx 18.4\text{ cm}^2$$

④ The area of the triangle is about 18.4 cm^2.

Objective: Find the Area of Non-Right Triangles

Practice) Find the area of the triangle. Round to the nearest tenth.



The area of the triangle is about 49.2 square inches.

1. *find* $m\angle C$

$$\frac{\sin C}{9} = \frac{\sin 57^\circ}{11.1}$$

$$m\angle C = \sin^{-1}\left(\frac{9\sin 57^\circ}{11.1}\right)$$

$$m\angle C \approx 42.84^\circ$$

2. *find* $m\angle B$

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$57^\circ + m\angle B + 42.84^\circ = 180^\circ$$

$$m\angle B \approx 80.16^\circ$$

3. find the area using: $A = \frac{1}{2}ac\sin B$

$$A \approx \frac{1}{2}(11.1in)(9in)\sin 80.16^\circ$$

$$A \approx 49.2\,in^2$$

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Closure

Describe the relationship between the angle and the two sides of the triangle in the area formula $A = \frac{1}{2}ab\sin C$.

The angle, C, is the included angle, which is the angle between the two sides of the triangle, a and b.