Objective: Find the domain of functions with rational exponents.

## Concept

### Rational Exponents

$$\boxed{b^{\frac{1}{n}} = \sqrt[n]{b}}$$

and

$$\boxed{b^{\frac{p}{n}} = \sqrt[n]{b^p} \text{ or } b^{\frac{p}{n}} = \left(\sqrt[n]{b}\right)^p}$$

### Negative Exponent Property

$$a^{-n} = \frac{1}{a^n} \quad or \quad \frac{1}{a^{-n}} = a^n$$

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Concept

Negative exponents can be used to write rational functions.

For example, $f(x) = x^{-1}$ is the same as $f(x) = \frac{1}{x}$, the parent function for rational functions.

Recall, the domain of a rational function must be values of the independent variable, such as $x$, that make the function defined. Any value of the independent variable that makes the function undefined (creates division by zero) or creates imaginary values is excluded from the domain.

Recall, the domain of a square root function is only values of the independent variable, such as $x$, that create non-negative radicands, so that the range will be in the set of real numbers. $(radicand \geq 0)$.

Objective: Find the domain of functions with rational exponents.

Ex) 1 Rewrite the function in radical form and find the domain of the function. Write the domain in set and interval notation.

$$f(x) = \frac{3}{4}x^{-\frac{2}{3}}$$

1

$$f(x) = \frac{3}{4} \cdot \frac{1}{x^{2/3}}$$

$$f(x) = \frac{3}{4x^{2/3}}$$

$$\boxed{f(x) = \frac{3}{4\sqrt[3]{x^2}}}$$ radical form

2 domain

denominator $\neq 0$

$$\frac{4\sqrt[3]{x^2}}{4} \neq \frac{0}{4}$$

$$(\sqrt[3]{x^2})^3 \neq (0)^3$$

$$x^2 \neq 0$$

$$\sqrt{x^2} \neq \pm\sqrt{0}$$

$$x \neq 0$$

dom. dom.

0 $x$

set: $\{x \mid x < 0 \text{ or } x > 0\}$

interval $(-\infty, 0) \cup (0, \infty)$

domain

Objective: Find the domain of functions with rational exponents.

Ex) Rewrite the function in radical form and find the domain of the function. Write the domain in set and interval notation.

$$g(x) = \frac{5}{7}x^{-\frac{1}{2}}$$

① $g(x) = \frac{5}{7} \cdot \frac{1}{x^{1/2}}$

$\boxed{g(x) = \frac{5}{7\sqrt{x}}}$ radical form

② domain

den. $\neq 0$ and radicand $\geq 0$

$\frac{7\sqrt{x}}{7} \neq \frac{0}{7}$ $\qquad x \geq 0$

$(\sqrt{x})^2 \neq (0)^2$

$x \neq 0$

dom.

0 x

set: $\{x \mid x > 0\}$

interval: $(0, \infty)$

domain

Objective: Find the domain of functions with rational exponents.

Ex) Rewrite the function in radical form and find the domain of the function. Write the domain in set and interval notation.

$$p(t) = 3t^{-\frac{1}{3}} - 2$$

1

$$p(t) = \frac{3}{t^{1/3}} - 2$$

$$\boxed{p(t) = \frac{3}{\sqrt[3]{t}} - 2}$$

radical form

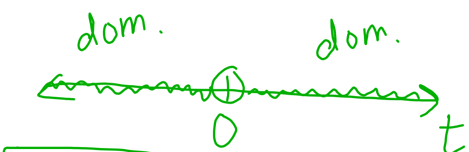
2 domain

den. $\neq 0$

$$\sqrt[3]{t} \neq 0$$

$$\left(\sqrt[3]{t}\right)^3 \neq 0^3$$

$$t \neq 0$$




domain

set $\{t \mid t < 0 \text{ or } t > 0\}$

interval $(-\infty, 0) \cup (0, \infty)$

Objective: Find the domain of functions with rational exponents.

Ex) Rewrite the function in radical form and find the domain of the function. Write the domain in set and interval notation.

$$v(t) = 8t^{-\frac{1}{2}} + 3$$

1

$$v(t) = \frac{8}{t^{1/2}} + 3$$

$$\boxed{v(t) = \frac{8}{\sqrt{t}} + 3}$$ radical form

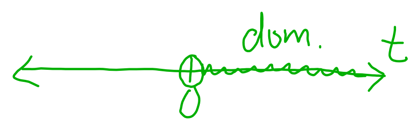
2 domain

den. $\neq 0$ and radicand $\geq 0$

$\sqrt{t} \neq 0$ $\quad$ $t \geq 0$

$t \neq 0$

$\boxed{\text{set: } \{t \mid t > 0\} \quad \text{interval } (0, \infty)}$



Objective: Find the domain of functions with rational exponents.

Ex) Rewrite the function in radical form and find the domain of the function. Write the domain in set and interval notation.

$$f(x) = x(x^2 - 25)^{-\frac{1}{2}}$$

1

$$\boxed{f(x) = \frac{x}{\sqrt{x^2-25}}}$$ radical form

2 domain.

den. $\neq 0$ and radicand $\geq 0$

$\sqrt{x^2-25} \neq 0$

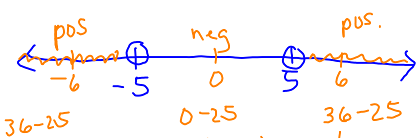
$x^2-25 \neq 0$

$x^2 \neq 25$

$\sqrt{x^2} \neq \pm\sqrt{25}$

$x \neq \pm 5$

$x^2-25 \geq 0$




test intervals $x^2-25$

set: $\{x \mid x < -5 \text{ or } x > 5\}$

interval: $(-\infty, -5) \cup (5, \infty)$

domain

Objective: Find the domain of functions with rational exponents.

Ex) Rewrite the function in radical form and find the domain of the function. Write the domain in set and interval notation.

$$g(x) = 2(x-1)(x^2+1)^{-\frac{1}{2}}$$

1 $\boxed{g(x) = \dfrac{2(x-1)}{\sqrt{x^2+1}}}$ radical form

2 domain

den. $\neq 0$

$\sqrt{x^2+1} \neq 0$

$x^2+1 \neq 0$

$x^2 \neq -1$

$\sqrt{x^2} \neq \pm\sqrt{-1}$

$x \neq \pm i$

radicand $\geq 0$

$x^2+1 \geq 0$

domain is the set of all real numbers

set: $\{x \mid -\infty < x < \infty\}$

interval $(-\infty, \infty)$

Objective: Find the domain of functions with rational exponents.

Ex) Rewrite the function in radical form and find the domain of the function. Write the domain in set and interval notation.

$$v(t) = (t^2 - 9)^{-\frac{1}{3}}$$

1 $\boxed{v(t) = \dfrac{1}{\sqrt[3]{t^2-9}}}$ radical form

2 domain

den. $\neq 0$

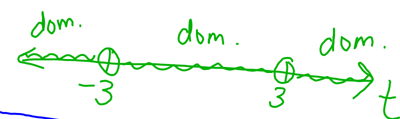
$\sqrt[3]{t^2-9} \neq 0$

$t^2 - 9 \neq 0$

$t^2 \neq 9$

$\sqrt{t^2} \neq \pm\sqrt{9}$

$t \neq \pm 3$




set: $\{t \mid t < -3 \text{ or } -3 < t < 3 \text{ or } t > 3\}$

interval $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$