Objective: Fit Exponential Functions to Data

Concept

Basic Exponential Function Model

An exponential function with the x-axis as its horizontal asymptote can be written in the form $f(x) = a(b)^x$, where a is the y-intercept and b is the common growth/decay factor.

A Test to Determine if Data is Exponential

1. The x values must increase in equal increments.
2. Determine the ratios of successive y values. For $x_2 > x_1$, then do $\frac{y_2}{y_1}$. This will ensure you determine the correct growth factor or decay factor for the function.
3. If the ratios of successive y values are constant, then the function is exponential. If the ratio is greater than 1, the function is exponential growth. If the ratio is between 0 and 1, the function is exponential decay.

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Ex) ① Is the data in the table exponential? ② If so, write the exponential function model.

x	0	1	2	3	4
$f(x)$	9	27	81	243	729

① $\frac{27}{9} = 3$ $\frac{81}{27} = 3$ $\frac{243}{81} = 3$ $\frac{729}{243} = 3$

The data is exponential.

② model: $f(x) = a(b)^x$

ⓐ base = $b = 3$ (growth factor $b = 3 > 1$)

$f(x) = a(3)^x$

ⓑ $a = ?$ use $(0, 9)$

$9 = a(3)^0$

$9 = a \cdot 1$

$9 = a$

③ final model $f(x) = 9(3)^x$

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Practice) Is the data in the table exponential? If so, write the exponential function model.

x	0	1	2	3	4
$f(x)$	256	64	16	4	1

$$\frac{64}{256}=\frac{1}{4} \quad \frac{16}{64}=\frac{1}{4} \quad \frac{4}{16}=\frac{1}{4} \quad \frac{1}{4}=\frac{1}{4}$$

The data is the table is exponential.

$$b=\frac{1}{4}$$

$$f(x)=a(b)^x \rightarrow f(x)=a\left(\frac{1}{4}\right)^x$$

$$a=? \rightarrow use(0,256) \rightarrow 256=a\left(\frac{1}{4}\right)^0 \rightarrow 256=a$$

$$\text{final model}: f(x)=256\left(\frac{1}{4}\right)^x$$

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Creating an Exponential Model to Fit Data

In the real world, sets of data rarely fit a model perfectly, but if the ratios of successive values are approximately equal, an exponential model can still be a good model.

To create an exponential model of the form $f(x) = a(b)^x$, values for a and b need to be determined using the data.

Steps:

1. Find the base (growth/decay factor) b by averaging the ratios of successive values of $f(x)$.

2. Find a using the data point with an x value of 0.

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Ex) The data give the estimated value in dollars of a model of classic car over several years. Find an approximate exponential model for the car's value.

Year	1992	1993	1994	1995	1996	1997	1998
	0	1	2	3	4	5	6
Value	15,300	16,100	17,300	18,400	19,600	20,700	22,000

① let t = years since 1992

V = the car's estimated value in dollars

② find b (base)

ⓐ $\frac{y_2}{y_1} = \frac{16100}{15300} \approx 1.052$ $\quad \frac{17300}{16100} \approx 1.075$ $\quad \frac{18400}{17300} \approx 1.064$ $\quad \frac{19600}{18400} \approx 1.065$

$\frac{20700}{19600} \approx 1.056$ $\quad \frac{22000}{20700} \approx 1.063$

ⓑ average

$$b \approx \frac{1.052 + 1.075 + 1.064 + 1.065 + 1.056 + 1.063}{6}$$

$b \approx 1.0625$

ⓒ model: $V(t) = a(b)^t \rightarrow V(t) = a(1.0625)^t$

③ find a.

use: (0 yr, \$15300) $\rightarrow 15300 = a(1.0625)^0$

$15300 = a$

④ final model: $V(t) = 15300(1.0625)^t$

where t is years since 1992 and V is the car's estimated value in dollars.

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Practice) The total catch in millions of tons of Iceland's fisheries from 2002 to 2010 is shown in the table. Create an approximate exponential model for the data set.

	Year	Total Catch (Millions of Tons)
0	2002	2.145
1	2003	2.002
2	2004	1.750
3	2005	1.661
4	2006	1.345
5	2007	1.421
6	2008	1.307
7	2009	1.164
8	2010	1.063

Step 1: Define the variables

let t = years since 2002

let $C(t)$ be the total catch in millions of tons

Step 2: Find the value for b. Average the successive ratios of the total catch.

$$b = \frac{0.933 + 0.874 + 0.949 + 0.809 + 1.057 + 0.920 + 0.891 + 0.913}{8}$$

$b = 0.91825 \approx 0.918$

Step 3: Find the value for a.

$when\ t = 0, C = 2.145$

$$C(t) = a(b)^t$$
$$2.145 = a(0.918)^0$$
$$2.145 = a$$

Step 4: Write the exponential model.

$C(t) = 2.145(0.91825)^t$, where t is years since 2002 and $C(t)$ is the total catch in millions of tons

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Closure

Once you create an exponential model, how can you test its accuracy?

Once I create an exponential model I can test its accuracy by plugging the values from one of the data points into the model and verifying the two sides of the equation result in values that are close to one another.