Objective: Solve work problems using a rational equation.

Concept

$$\text{Work} = \text{Rate} \cdot \text{Time}$$

Work = the number of jobs to be completed (1 job = 1 work)

Rate = how fast the work can be done (always a ratio)

Time = how long someone is working to complete the job(s)

**Solving the equation Work = Rate · Time for Rate yields the following definition:

$$Rate = \frac{Work}{Time} = \frac{1\ job\ completed}{time\ to\ complete\ the\ job}$$

We will use this definition to write expressions for Rate in what are often called "Work Problems."

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Concept

First, we are going to learn how to write expressions for Rate.

1. If Carlos can mow the back lawn in 30 minutes, what is his rate?

$$Rate = \frac{1\ lawn\ mown}{30\ minutes}$$

$$This\ also\ means: an\ average\ of\ \frac{1}{30}\ of\ the\ lawn\ is\ mowed\ each\ minute$$

2. Samantha can paint a new shed in 5 hours. What is the rate at which she paints the shed?

$$Rate = \frac{1\ shed\ painted}{5\ hours}$$

$$This\ also\ means: an\ average\ of\ \frac{1}{5}\ of\ the\ shed\ is\ painted\ each\ hour$$

3. If Thomas can run a mile in 7.2 minutes, what is his rate?

$$Rate = \frac{1\ mile\ run}{7.2\ minutes}$$

$$This\ also\ means: an\ average\ of\ about\ 0.14\ of\ the\ mile\ is\ run\ each\ minute$$

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job/work

Ex) Water flowing through both a small pipe and a large pipe can fill a water tank in 9 hours. Water flowing through the large pipe alone can fill the tank in 17 hours. How much time, *t*, in minutes, would it take to fill the tank using only the small pipe?

Step 1: A table can be used to organize the information needed to write an equation.

	Rate
Large Pipe	$\frac{1 \text{ tank}}{17 \text{ hr}}$
Small Pipe	$\frac{1 \text{ tank}}{t \text{ hrs}}$
Working Together	$\frac{1 \text{ tank}}{9 \text{ hrs}}$

$\text{Rate} = \frac{\text{Work}}{\text{Time}}$

Step 2: Write an equation. Then solve the equation and answer the question.

large pipe rate + small pipe rate = working together rate

$$\frac{1}{17} + \frac{1}{t} = \frac{1}{9}$$

Solve

LCD = 17·9·t

$$\frac{1}{17}\cdot\frac{17\cdot9\cdot t}{1} + \frac{1}{t}\cdot\frac{17\cdot9\cdot t}{1} = \frac{1}{9}\cdot\frac{17\cdot9\cdot t}{1}$$

$$9t + 153 = 17t$$
$$-9t \qquad -9t$$
$$153 = 8t$$
$$\frac{153}{8} = \frac{8t}{8}$$

$t = 19.125$ hrs

convert

$$\frac{19.125 \text{ hr}}{1}\cdot\frac{60 \text{ min}}{1 \text{ hr}}$$

$= 1147.5$ min

conclusion

Using only the small pipe, it would take 1147.5 minutes to fill the tank.

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Practice) Kevin can clean a large aquarium tank in about 7 hours. When Kevin and Lara work together, they can clean the tank in 4 hours. How long would it take, in hours and minutes, for Lara to clean the tank if she worked by herself?

	Rate
Kevin	$\frac{1}{7}$
Lara	$\frac{1}{t}$
Working Together	$\frac{1}{4}$

Kevin's work + Lara's work = 1 clean tank

$$\frac{1}{7}+\frac{1}{t}=\frac{1}{4}$$

$$\frac{1}{7}\cdot(28t)+\frac{1}{t}\cdot(28t)=\frac{1}{4}\cdot(28t)$$

$$4t+28=7t$$

$$28=3t$$

$$t=\frac{28}{3}=9\frac{1}{3}\ hours=9\ hours\ 20\ \text{minutes}$$

$$\left(\frac{1}{3}\ hour\cdot\frac{60\ \text{min}}{1\ hour}=20\ \text{min}\right)$$

If Lara worked by herself, it would take her 9 hours and 20 minutes to clean the tank.

Objective: Solve work problems using a rational equation.

Ex) Nathan can paint a room (work/job) in 8 hours. His brother John would need 12 hours to paint the same room. If they work together, how long will it take, in hours and minutes, to paint the room?

$Rate_{Nathan}$	+ $Rate_{John}$	= $Rate_{together}$
$\frac{1\text{ room}}{8\text{ hr}}$	$\frac{1\text{ room}}{12\text{ hr}}$	$\frac{1\text{ room}}{t\text{ hr}}$

$$\frac{1}{8} + \frac{1}{12} = \frac{1}{t}$$

4·2 4·3

solve.

$LCD = 4 \cdot 2 \cdot 3 \cdot t$

$= 24t$

$$\frac{1}{8} \cdot \frac{24t}{1} + \frac{1}{12} \cdot \frac{24t}{1} = \frac{1}{t} \cdot \frac{24t}{1}$$

$$3t + 2t = 24$$

$$\frac{5t}{5} = \frac{24}{5}$$

$t = 4\frac{4}{5}$ or 4.8 hr

$$\cdot \frac{8\text{ hr}}{1} \cdot \frac{60\text{ min}}{1\text{ hr}} = 48\text{ min}$$

conclusion

If the brothers work together, it will take 4 hours 48 minutes to paint the room.

Objective: Solve work problems using a rational equation.

Practice) One incinerator can process a day's garbage in 14 hours. When the first incinerator is broken, a second incinerator can process a day's garbage in 20 hours. If both incinerators are working, how many hours, to the nearest tenth, will it take to process a day's garbage?

$Rate_{Incinerator\ 1}$	$Rate_{Incinerator\ 2}$	$Rate_{together}$
$\frac{1}{14}$	$\frac{1}{20}$	$\frac{1}{t}$

Incinerator 1's rate + Incinerator 2's rate = rate together

$$\frac{1}{14}+\frac{1}{20}=\frac{1}{t}$$

$$\frac{1}{14}\cdot(140t)+\frac{1}{20}\cdot(140t)=\frac{1}{t}\cdot(140t)$$

$$10t+7t=140$$

$$17t=140$$

$$t\approx 8.2\,hours$$

If both incinerators are working it will take about 8.2 hours to process a day's garbage.

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Closure

Explain how you can recognize when you are reading a "Work Problem."

A "Work Problem" can be recognized because the situation describes an activity being completed by two or more people/objects in a certain amount of time.