Objective: Solve problems in context using exponential models.

Concept

To identify an exponential function, $f(x) = a(b)^{cx}$, as growth or decay you should <u>consider the function when it is in the form where the coefficient of x is positive</u>. Then, if b is greater than 1, the function is exponential growth, and if b is between 0 and 1, the function is exponential decay. Another option, using a calculator, is to determine the value of b^c, because this is the actual value of the function's base.

In some exponential functions, especially those that model real-world situations, looking at the base alone can lead to a misinterpretation of the model.

For example: $r(h) = 3(2.4)^{-h}$ **is exponential decay** because the negative exponent means the base value is really $\frac{1}{2.4}$, which is $\frac{5}{12}$, a value between 0 and 1. The model can be rewritten as $r(h) = 3\left(\frac{5}{12}\right)^h$, and from this form the true base value is shown.

Objective: Solve problems in context using exponential models.

Concept

When solving context problems involving a function model, it is important to determine how the variables are defined, which variable value is needed to answer the question(s), and know which variable any given quantity represents.

A Strategy for Solving Context Problems

1. Determine how all variables in the function model are defined. Pay attention to and make note of units.
2. Determine which variable value is needed to answer the question. This will be the value you solve for.
3. Determine what any given values (quantities) represent in terms of the variables.
4. Substitute any given quantities into the function model and then solve the equation.
5. Make any necessary conclusions.

Objective: Solve problems in context using exponential models.

Ex) The decay of Sodium-22 can be modeled by the function $S(t) = ae^{-0.267t}$, where t is in years.

$e \approx 2.718$

a) Is the model exponential growth or exponential decay? Explain your reasoning.

base $= e^{-0.267} = \frac{1}{e^{0.267}} < 1 \rightarrow$ decay

The model is exponential decay because the base value is $e^{-0.267}$ which is less than 1.

b) If 300 grams of Sodium-22 was present in a soil sample in 2010, how much Sodium-22 will remain in 2020? Round to three decimal places.

model $S(t) = ae^{-0.267t}$

$a = 300$

$S(t) = 300e^{-0.267t}$

$S(t) = ?$ when $t = 2020 - 2010 = 10$ yr

$S(10) = 300e^{-0.267(10)} \approx 20.776$ grams

calc. 300 • 2nd [LN] e^(−0.267×10) =

About 20.776 grams of a 300 gram sample of Sodium-22 will remain in 2020.

Objective: Solve problems in context using exponential models.

Practice) The function $B(h) = ae^{0.2357h}$ models a bacteria colony in a favorable environment, where h is in hours.

a) Is the model exponential growth or exponential decay? Explain your reasoning.

The model is exponential growth because $e^{0.2357}$ is greater than 1. The approximate value is about 1.266.

b) If there are 750 bacteria when the scientist first counts the colony, how many bacteria will there be after 9 hours?

1. $a = 750$ (the initial value, when $h = 0$)

2. completed model: $B(h) = 750e^{0.2357h}$

3. $t = 9\ hours$

4. $B(9) = 750e^{0.2357(9)} \approx 6256\ bacteria$

After 9 hours the colony of 750 bacteria will have grown to be about 6256 bacteria.

Objective: Solve problems in context using exponential models.

Practice) A biologist studying a bird population that has been classified as endangered has determined that the population can be modeled by the function $B(t) = 4.8(3.2)^{0.04t}$, where t is years since the population started to be monitored in 2006 and $\mathrm{B}(t)$ is in thousands.

a) According to the model, is the bird population increasing or decreasing? Explain your reasoning.

According to the model, the bird population is increasing because the base of the exponential model is $3.2^{0.04}$ which is greater than 1, making it an exponential growth model.

b) According to the model, what is the bird population expected to be in 2020?

1. $t = 2020 - 2006 = 14\ years$
2. $B(14) = 4.8\left(3.2\right)^{0.04(14)} \approx 9.2071501...$
3. $\approx 9.2071501... \cdot 1000 \approx 9207\ birds$

According to the model, in 2020, the bird population is expected to be 9207.

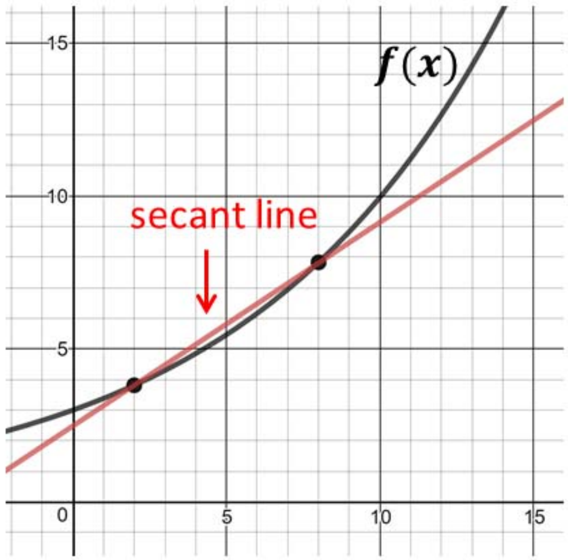
Objective: Solve problems in context using exponential models.

Concept

$$\text{Average Rate of Change } = \frac{\triangle f(x)}{\triangle x}$$

$$\frac{\triangle f(x)}{\triangle x} = \frac{\text{change in } f(x) \text{ values}}{\text{change in } x \text{ values}} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \text{ for the interval } [x_1, x_2]$$

- The **average rate of change** is the average change between y values for each unit of x over a specific interval.
- The **average rate of change** for an interval **corresponds to the slope of the line through the two points at the ends of the interval**. This line is called the **secant line**.



Objective: Solve problems in context using exponential models.

Ex) A chemist studying the effect of a new insecticide has determined the mathematical model for an insect population treated by the chemical as $P(d) = 216.8(1.8)^{-2.56d}$, where d is days since the application of the product, and P is in thousands. What is the average rate at which the insect population is changing per day from 1 to 4 days after application? Round to three decimal places.

① $AROC = \dfrac{\Delta P \text{ insects}}{\Delta d \text{ days}}$

② $d = 4$, $d = 1$

$P(4) = ?$ $P(1) = ?$

$\dfrac{\Delta P \text{ insects}}{\Delta d \text{ days}} = \dfrac{[P(4) - P(1)] \times 1000 \text{ insects}}{(4-1) \text{ days}}$

^(-2.56×4)

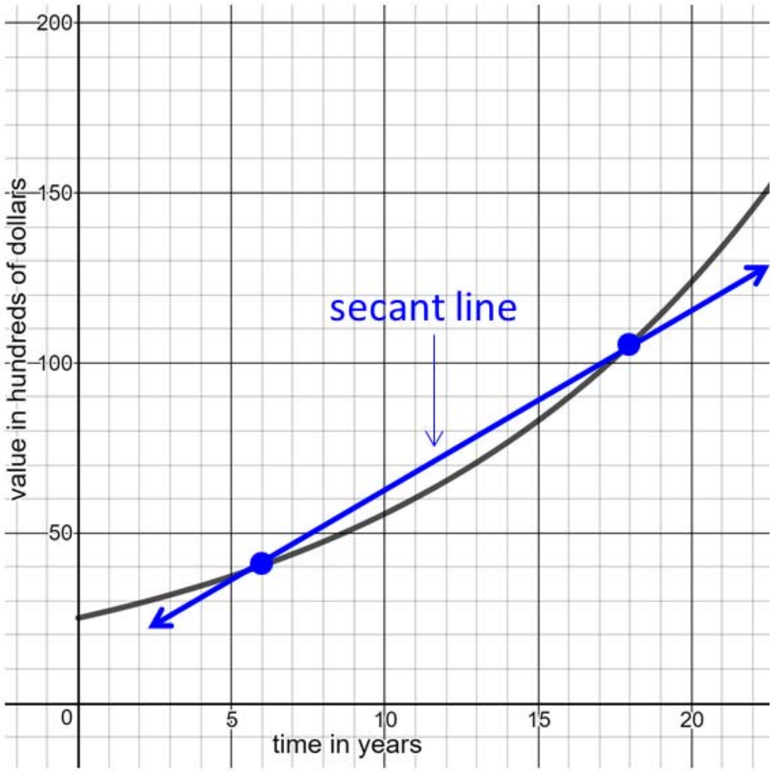
$= \dfrac{(216.8(1.8)^{-2.56(4)} \times 1000 - 216.8(1.8)^{-2.56(1)} \times 1000) \text{ insects}}{3 \text{ days}}$

$\approx -15{,}872.949$ insects per day

③ From 1 to 4 days after application, the insect population is decreasing at an average of about 15,872.949 insects per day.

Objective: Solve problems in context using exponential models.

Practice) The stock value of a company over each year since 1995 is shown in the graph. Estimate the average rate at which the stock value is increasing per year, to the nearest dollar, from 2001 to 2013?



1. *find year values*

$2013 - 1995 = 18\ years$

$2001 - 1995 = 6\ years$

2. *find corresponding stock values*

$for\ 6yr \rightarrow \$40 \times 100 = \$4000$

$for\ 18yr \rightarrow \$10.5 \times 100 = \$10,500$

$(6\ yr, \$4000)\ (18\ yr, \$10,500)$

3. *calculate AROC*

$$\frac{\triangle value}{\triangle years} \approx \frac{(\$10,500 - \$4000)}{(18-6)\ years} \approx \$542 \text{ per year}$$

The stock value increased at an average of about $542 per year from 2001 to 2013.

Objective: Solve problems in context using exponential models.

Practice) The table shows the total catch in millions of tons for Iceland's fisheries from 2002 to 2010. Estimate the average change in the total catch per year from 2005 to 2009.

Year	Total Catch (Millions of Tons)
2002	2.145
2003	2.002
2004	1.750
2005	1.661
2006	1.345
2007	1.421
2008	1.307
2009	1.164
2010	1.063

$(2005, 1.661\ million\ tons)$
$(2009, 1.164\ million\ tons)$

$$\frac{change\ in\ total\ catch}{change\ in\ time}$$

$$= \frac{(1.164\ million - 1.661\ million)\ tons}{(2009 - 2005) years}$$

$$= \frac{-0.497\ million\ tons}{4\ years} =$$

$$\frac{-0.497 \times 1{,}000{,}000\ \ tons}{4\ years} = -124{,}250\ tons\ per\ year$$

The total catch decreased at an average of 124,250 tons per year from 2005 to 2009.

Objective: Solve problems in context using exponential models.

Closure

Is the procedure for calculating the average rate of change over an interval for an exponential function the same as or different from the way average rate of change is calculated for other functions, such as linear and quadratic? Explain your reasoning.

The procedure is the same because the average rate of change formula, $\frac{\Delta f(x)}{\Delta x} = \frac{f(b)-f(a)}{b-a}$ over the interval $[a, b]$ is the same for all functions.