Objective: Find domain and average rate of change of a rational function

Concept

The domain of a rational function includes all real numbers except where the function is said to be undefined. A rational function is undefined for any value of x that makes the denominator equal to 0. These values of x also correspond to where the function has holes or vertical asymptotes.

To find the domain of a function $f(x)$ algebraically:

1. **Determine the values of x for which the function is undefined.** Set the denominator equal to 0 and solve the equation for all values of x. These will be the values excluded from the domain of the function.

2. Write the domain using the specified notation.

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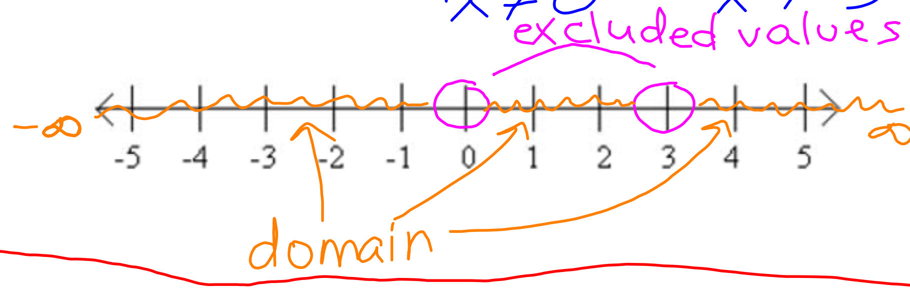
Ex) Find the domain of the function. Use both inequality and interval notation.

$$f(x) = \frac{3x}{x^2 - 3x} \longrightarrow x^2 - 3x \neq 0$$

$$x(x-3) = 0$$

$$x = 0 \qquad x - 3 = 0$$

$$x \neq 0 \qquad x \neq 3$$



domain

inequality $x < 0$ or $0 < x < 3$ or $x > 3$

interval $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$

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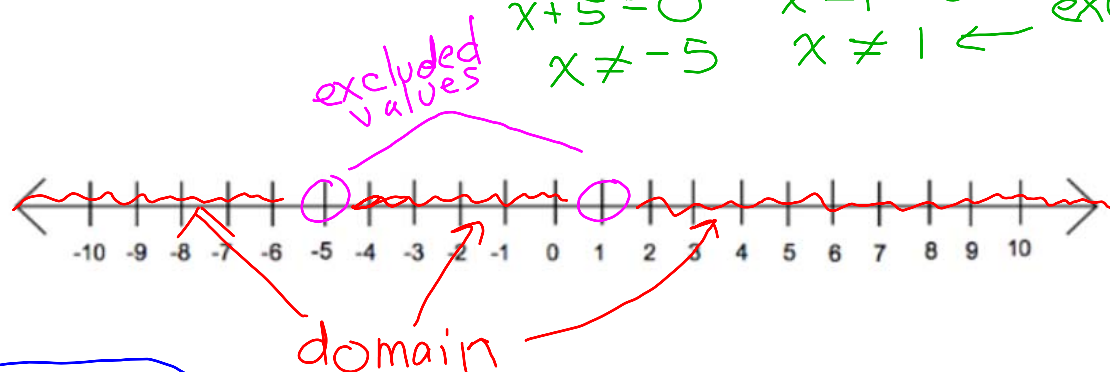
Ex) Find the domain of the function. Use both inequality and interval notation.

$$f(x) = \frac{x+5}{x^2+4x-5} \rightarrow x^2+4x-5 \neq 0$$

$(x+5)(x-1)=0$

$x+5=0$ $\quad$ $x-1=0$

$x \neq -5$ $\quad$ $x \neq 1$ ← excluded values



domain

inequality $x < -5$ or $-5 < x < 1$ or $x > 1$

interval $(-\infty, -5) \cup (-5, 1) \cup (1, \infty)$

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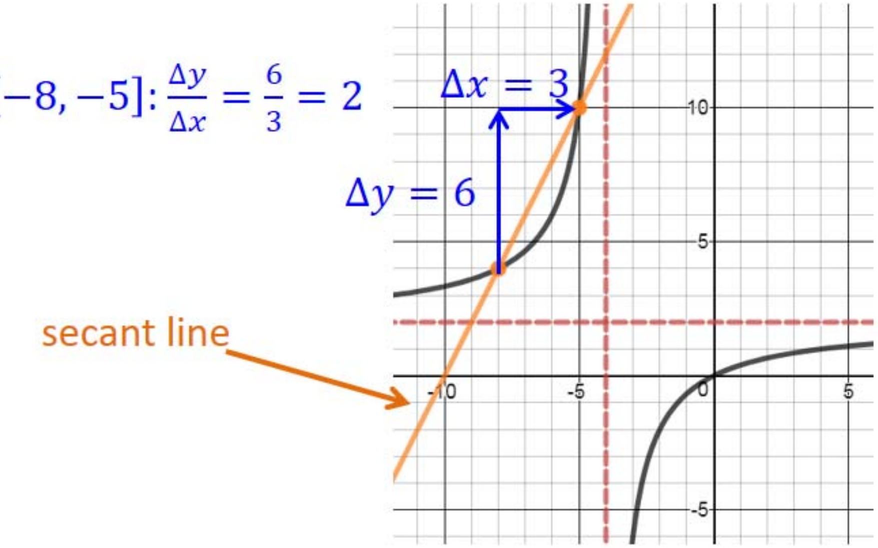
Concept

Average Rate of Change

The **average rate of change** is the average change between y values for each unit of x over a specific continuous interval of a function.

The **average rate of change** for an interval **corresponds to the slope of the line through the two points at the ends of the interval.** This line is called the **secant line.**

Average rate of change over $[-8, -5]$: $\frac{\Delta y}{\Delta x} = \frac{6}{3} = 2$



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Concept

To find the average rate of change of a function $f(x)$ over a continuous interval $[x_1, x_2]$:

1. Calculate the corresponding function values for x_1 and x_2. (i.e. Find $f(x_1)$ and $f(x_2)$.)

2. Calculate the average rate of change over the interval: $\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

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Ex) The function $D = \frac{50}{m}$ represents the density, in cm^3/g, of an object with a volume of 50 cubic centimeters and a mass of m grams. Find the average change in the object's density from a mass of 5 to 16 grams.

① function $D = \frac{50}{m}$

② interval $[5, 16]$ grams

$[x_1, x_2]$

$[m_1, m_2]$

③ AROC $\frac{\Delta D}{\Delta m} = \frac{D(16) - D(5)\ cm^3/g}{16 - 5\ \text{grams}}$

$= \frac{\left(\frac{50}{16} - \frac{50}{5}\right) cm^3/g}{(16 - 5)\ \text{grams}}$

$= -0.625\ cm^3/g$ per gram of mass

The average change in the object's density from 5 to 16 g is a decrease of $0.625\ cm^3/g$ per gram of mass.

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Closure

Whenever you are finding an average rate of change for a specific interval of a function, the calculation is the same as the ____slope____ of the secant line.

The mathematical formula is $\frac{\Delta f}{\Delta x} = \frac{f(x_2)-f(x_1)}{x_2-x_1}$ or $\frac{y_2-y_1}{x_2-x_1}$.