Objective: Solve river current problems using rational equations.

Concept

Distance = Rate · Time

Distance = how far something travels (7 miles, 11 kilometers, 5 feet)

Rate = how fast something travels (always a ratio: 45 mph, 17 km/h, 12 ft/sec)

Time = how long something is traveling (2 hours, 3 minutes, 30 seconds)

**Solving the equation Distance = Rate · Time for Time yields the following definition:

$$Time = \frac{Distance}{Rate} = \frac{how\ far\ something\ travels}{how\ fast\ something\ travels}$$

We will use this definition to write expressions for Time in what are often called "Motion Problems or River Current Problems."

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Upstream means something is moving against the direction the water is moving, so the water is moving toward the object. In this case, **the push of the water will slow the object down**.

Object →← Water

$$\textit{Rate of Object Upstream} = \textit{Object's Speed} - \textit{Water's Speed}$$

Downstream means something is moving in the same direction the water is moving, or with the water. In this case, **the push of the water will make the object move faster than it can go on its own**.

Water →Object →

$$\textit{Rate of Object Downstream} = \textit{Object's Speed} + \textit{Water's Speed}$$

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Ex) Abram rows 9 miles downstream in his canoe and then 3 miles upstream to find a hiking trail. The trip takes him 2.3 hours. If Abram's average speed in still water is about 5 miles per hour, how fast is the river's current? Round to the nearest tenth if necessary.

c

Step 1: A table can be used to organize the information needed to write an equation.

	Distance	Rate	Time
upstream	3 mi	$5-c$	$\frac{3}{5-c}$
downstream	9 mi	$5+c$	$\frac{9}{5+c}$

Step 2: Use the Time column to write an equation. Then solve the equation and answer the question.

time upstream + time downstream = total travel time

$$\frac{3}{5-c} + \frac{9}{5+c} = 2.3$$

$LCD = (5-c)(5+c)$

$$3(5+c) + 9(5-c) = 2.3(5-c)(5+c) \quad (25-c^2)$$

$$15 + 3c + 45 - 9c = 57.5 - 2.3c^2$$

$$2.3c^2 - 6c + 2.5 = 0$$

$$c = \frac{-1b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$c = \frac{-1(-6) \pm \sqrt{(-6)^2 - 4(2.3)(2.5)}}{2(2.3)}$$

$$c = \frac{6 \pm \sqrt{13}}{4.6}$$

$$c = \frac{(6+\sqrt{13})}{4.6} \approx 2.1 \text{ mph}$$

$$c = \frac{(6-\sqrt{13})}{4.6} \approx 0.5 \text{ mph}$$

Conclusion

The rate of the river's current is about 2.1 mph or about 0.5 mph.

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Ex) A riverboat travels at an average of 14 kilometers per hour in still water. The riverboat travels 110 km up the Ohio River and 110 km down the same river in 17.5 hours. To the nearest tenth of a kilometer per hour, what was the average speed of the current (c) of the river? Round to the nearest tenth if necessary.

	Distance	Rate	Time
upstream	110 km	$14-c$	$\frac{110}{14-c}$
downstream	110 km	$14+c$	$\frac{110}{14+c}$

$$\frac{110}{14-c} + \frac{110}{14+c} = 17.5$$

$LCD = (14-c)(14+c)$

$$110(14+c) + 110(14-c) = 17.5(14-c)(14+c)$$

$$1540 + \cancel{110c} + 1540 - \cancel{110c} = 3430 - 17.5c^2$$

$$\frac{17.5c^2}{17.5} = \frac{350}{17.5}$$

$$c^2 = 20$$

$$\sqrt{c^2} = \pm\sqrt{20}$$

$c \approx 4.5$ km/hr

The average speed of the river's current is about 4.5 kilometers per hour.

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Closure

Does an object travel faster when moving upstream or downstream on a river? Explain your reasoning.

An object travels faster when moving downstream because the object is going the same direction as the water, so the push of the water from behind the object will increase its speed.