Objective: Solve Trigonometric Equations Using Double-Angle Identities.

Concept

Steps to Solve a Trigonometric Equation that Includes a Double-Angle

1. Use a Double-Angle Identity to rewrite the double angle expression.
2. Solve the Equation.
   Given a Quadratic Structure:
   1. Write the equation in standard form.
   2. Use a quadratic strategy.
      1. Square Root Property
      2. Factoring
      3. Quadratic Formula
3. Find the angle measure(s) that correspond to the function value(s).

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Ex) Solve $\sin 2x \cos x - \sin x = 0$ for $0 \le x < 2\pi$.

① $2\sin x\cos x \cdot \cos x - \sin x = 0$

$2\sin x\cos^2 x - \sin x = 0$ * quadratic structure

② $\sin x(2\cos^2 x - 1) = 0$

$\sin x = 0 \qquad 2\cos^2 x - 1 = 0$

$\cos^2 x = \frac{1}{2}$

$\sqrt{\cos^2 x} = \pm\sqrt{\frac{1}{2}}$

③ $x = 0, \pi$

$\cos x = \pm\frac{\sqrt{2}}{2}$

③ $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

solutions $\boxed{x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}}$

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Ex) Solve $\cos 2x + \cos x = 0$ for $[0, 2\pi)$.

① $2\cos^2 x - 1 + \cos x = 0$ quadratic struction

② $2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ $\cos x + 1 = 0$

$\cos x = \frac{1}{2}$ $\cos x = -1$

③ $x = \frac{\pi}{3}, \frac{5\pi}{3}$ $x = \pi$

solutions: $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

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Closure

Brandon needs to solve the equation $\cos 2x + 5\sin x = 4$. Which Double-Angle Identity should Brandon use? Explain your reasoning.

Brandon should use the Double-Angle Identity $1 - 2\sin^2 x$ so that the terms of the equation will all be powers of $\sin x$.

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Concept

Double-Angle Identities	
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
$\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$	$\cos 2\alpha = 1 - 2\sin^2\alpha$ $\cos 2\alpha = 2\cos^2\alpha - 1$