

Objective: Solve equations.

Concept

When solving an equation you must recognize the family it belongs to so you know what algebraic strategies/procedures to use and whether there may be extraneous or invalid solutions.

Type of Equation	Types of Solutions
<p><u>Linear</u> Example: $3x - 6 = 2x + 5$</p>	One solution, no solution, infinitely many solutions.
<p><u>Polynomial</u> Examples: $x^2 - 5x = 0$, $x^3 - 6x^2 + 5x = 0$</p>	The number of solutions is equal to the degree of the polynomial. Solutions can be real or imaginary numbers.
<p><u>Square Root</u> Example: $\sqrt{x - 1} = 7$</p>	Solutions must be real numbers and make the radicand ≥ 0 .
<p><u>Cube Root</u> Example: $\sqrt[3]{3x} = 4$</p>	Solutions must be real numbers.
<p><u>Rational</u> Examples: $\frac{3}{x} = \frac{2}{x+4}$, $\frac{4}{x} - \frac{3}{x-2} = 5$</p>	Solutions cannot make the denominator equal to 0.
<p><u>Exponential</u> Examples: $4 = 8^x$, $2e^x = 12$</p>	Solutions must be real numbers.



Objective: Solve equations.

Solve the inequality.

$$-x + 2 > 5$$

$$-x + 2 > 5$$

$$-x > 3$$

$$\frac{-x}{-1} > \frac{3}{-1}$$

$$x < -3$$



Objective: Solve equations.

Solve the equation.

$$9 + 7(x + 2) = 3x + 4(x - 4)$$

$$9 + 7x + 14 = 3x + 4x - 16$$

$$7x + 23 = 7x - 16$$

$$23 = -16$$

$$23 \neq -16 \rightarrow \text{no solution}$$



Objective: Solve equations.

Solve the equation.

$$x^2 - 9x = -20$$

$$x^2 - 9x + 20 = 0$$

$$(x-5)(x-4) = 0$$

$$x-5=0, x-4=0$$

$$x=5, x=4$$

Quadratic Formula

Given $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$$

Objective: Solve equations.

Solve the equation.

$$25x^2 - 36 = 0$$

$$(5x+6)(5x-6) = 0$$

$$5x+6 = 0, \quad 5x-6 = 0$$

$$x = -\frac{6}{5}, \quad x = \frac{6}{5}$$

$$25x^2 = 36$$

$$x^2 = \frac{36}{25}$$

$$x = \pm \sqrt{\frac{36}{25}} = \pm \frac{\sqrt{36}}{\sqrt{25}}$$

$$x = \pm \frac{6}{5}$$

Objective: Solve equations.

Solve the equation.

$$\frac{6}{x} = 9$$

$$\frac{6}{x} = \frac{9}{1}$$

$$9x = 6$$

$$x = \frac{6}{9}$$

$$x = \frac{2}{3}$$



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Solve the equation.

$$\frac{2}{5x} = \frac{4}{10x}$$

$$20x = 20x$$

$$0 = 0$$

infinitely many solutions

What value of x is not part of the infinite number of solutions? Why?

The value $x = 0$ is not a valid solution because division by 0 is undefined.



Objective: Solve equations.

Solve the equation.

$$\frac{7}{x^2} = \frac{1}{x}$$

$$x^2 = 7x$$

$$x^2 - 7x = 0$$

$$x(x-7) = 0$$

$$\cancel{x=0}$$

extraneous

$$x-7=0$$

$$x=7$$



Objective: Solve equations.

Solve the equation.

$$5\sqrt{x} = 45$$

$$\frac{5\sqrt{x}}{5} = \frac{45}{5}$$

$$\sqrt{x} = 9$$

$$(\sqrt{x})^2 = 9^2$$

$$\boxed{x = 81}$$



Objective: Solve equations.

Solve the equation.

$$\sqrt{x+2} + 8 = 2$$

$$\sqrt{x+2} = -6$$

$$(\sqrt{x+2})^2 = (-6)^2$$

$$x+2 = 36$$

$$~~x = 34~~ \rightarrow \boxed{\text{no solution}}$$

$$\text{check: } \sqrt{34+2} = -6$$

$$\sqrt{36} \neq -6$$

Objective: Solve equations.

Solve the equation.

$$\sqrt[3]{x} = 4$$

$$\left(\sqrt[3]{x}\right)^3 = (4)^3$$

$$x = 64$$



Objective: Solve equations.

Solve the equation.

$$2^x = 19$$

$$\log 2^x = \log 19$$

$$x \cdot \log 2 = \log 19$$

$$x = \frac{\log 19}{\log 2}$$

$$\ln 2^x = \ln 19$$

$$x \cdot \ln 2 = \ln 19$$

$$x = \frac{\ln 19}{\ln 2}$$

$$x = \log_2 19$$

logarithmic form



Objective: Solve equations.

Solve the equation.

$$9 = 12^x$$

$$\log 9 = \log 12^x$$

$$\log 9 = x \cdot \log 12$$

$$x = \frac{\log 9}{\log 12}$$

$$\ln 9 = \ln 12^x$$

$$\ln 9 = x \cdot \ln 12$$

$$x = \frac{\ln 9}{\ln 12}$$

$$x = \log_{12} 9$$

logarithmic form



Objective: Solve equations.

Solve the equation.

$$\frac{x}{x+3} = \frac{2}{7}$$

$$2(x+3) = 7x$$

$$2x+6 = 7x$$

$$6 = 5x$$

$$x = \frac{6}{5}$$

